Fundamentals of Road Construction

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Lecture 3

The subject of the lecture:

Horizontal alignment

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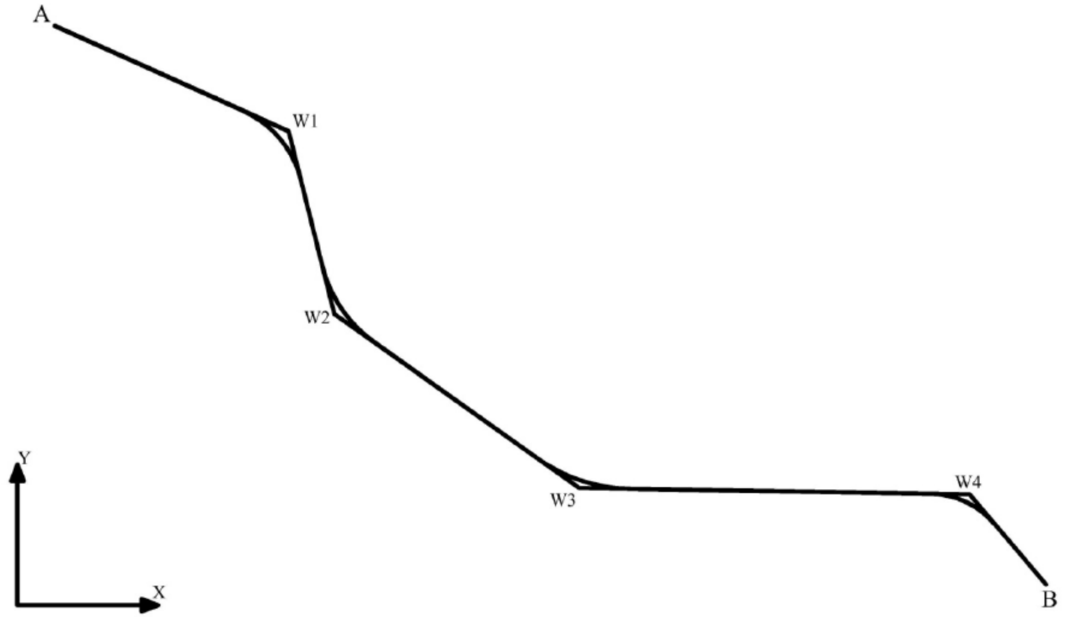
Coordinates of vertex points of horizontal alignment of the design road:



Point	Coordinates [m]	
	X	Y
A	90,00	1400,00
W_1	655,05	1145,05
W_2	765,65	704,15
W_3	1356,55	283,45
W_4	2300,07	268,08
B	2485,00	50,00

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Distance (length) between vertex points:

$$\overline{AW_1} = \sqrt{(X_{W1} - X_A)^2 + (Y_{W1} - Y_A)^2}$$

$$\overline{AW_1} = \sqrt{(655{,}05m - 90{,}00m)^2 + (1145{,}05m - 1400{,}00m)^2} = 619{,}90m$$

Section	Distance [m]
$\overline{AW_1}$	619,90
$\overline{W_1W_2}$	454,56
$\overline{W_2W_3}$	725,36
$\overline{W_3W_4}$	943,65
$\overline{W_4B}$	285,94
Σ	3029,41

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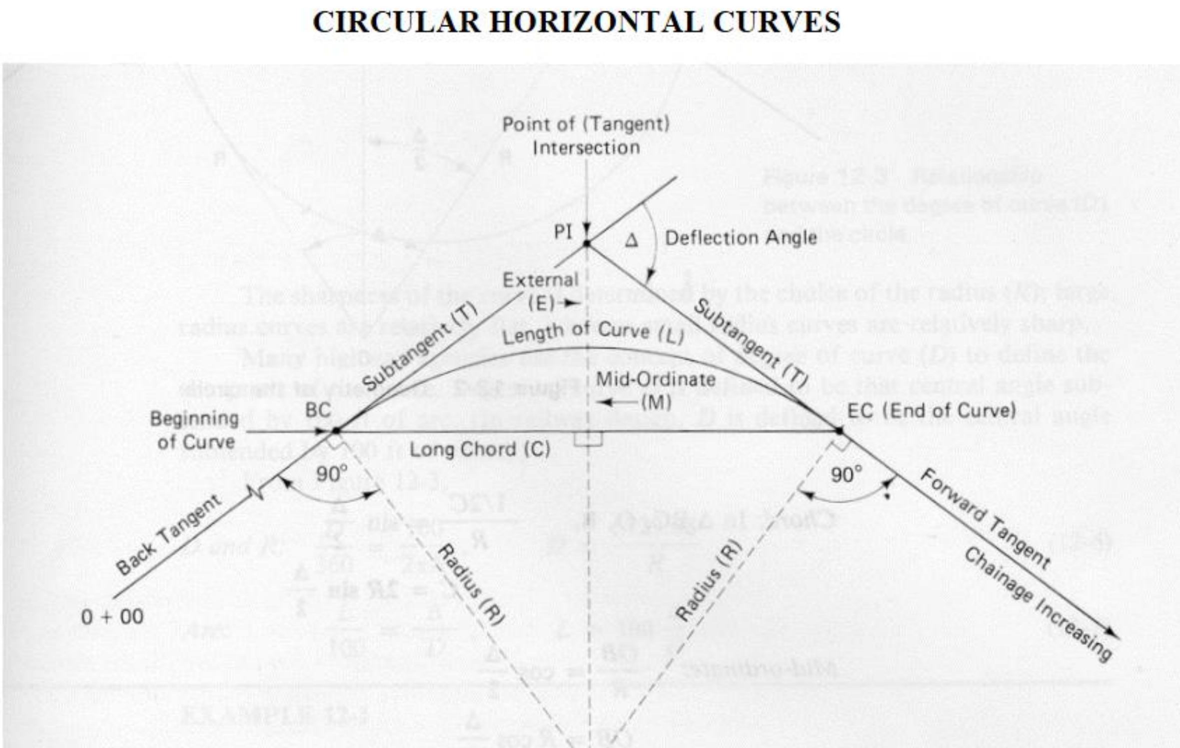
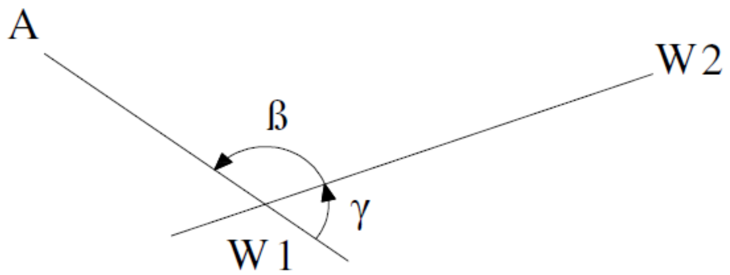
Deflection angle of horizontal alignment of the design road:



BC =Beginning of Curve	EC = End of Curve
PC = Point of Curve	PT = Point of Tangent
TC = Tangent to Curve	CT = Curve to Tangent

Source: https://www.cpp.edu/~hturner/ce220/circular_curves.pdf



$$\cos\beta_1 = \frac{\overline{AW_1}^2 + \overline{W_1W_2}^2 - \overline{AW_2}^2}{2 \cdot \overline{AW_1} \cdot \overline{W_1W_2}}$$

$$\gamma_1 = 180^\circ - \beta_1$$

$$\cos\beta_1 = \frac{(619{,}90)^2 + (454{,}56)^2 - (969{,}90)^2}{2 \cdot 619{,}90 \cdot 454{,}56} = -0{,}620696$$

$$\beta_1 = 128{,}3669^\circ$$

$$\gamma_1 = 180^\circ - 128{,}3669^\circ = 51{,}6331^\circ$$

	Deflection angle γ		
	[°]		[rad]
γ_1	51°37'59"	51,6331	0,901167
γ_2	40°28'06"	40,4684	0,706306
γ_3	34°30'59"	34,5163	0,602422
γ_4	48°46'09"	48,7691	0,851181

Tortuosity of section of the design road:

$$K = \frac{\sum_{i=1}^{n} |\gamma_n|}{L} \text{ [°/km], where:}$$

K - tortuosity of the section of the design road [°/km]
$\Sigma\gamma_n$ - sum of the absolute deflection angles of horizontal alignment [°]
L - distance between vertices [km]
n - number of vertices [-]

$$K = \frac{51{,}6331° + 40{,}4684° + 34{,}5163° + 48{,}7691°}{3{,}03} = \frac{175{,}3867°}{3{,}03}$$

$$K = 58 \text{ °/km}$$

Checking the requirements for the assumed radius of horizontal curves

The adoption of horizontal curve radius based on *regulation of the Minister of Transport and Maritime Economy (consolidated text, Journal of Laws of 2016, item 124, as amended) – in short JoL16.*

a) roll-over stability condition

$$R_{\min} = \frac{v^2}{g \cdot \left(\frac{b}{2h} \pm i_0 \right)}, \ [\mathrm{m}] \ \text{where:}$$

v- speed [m/s]

$$v = \begin{cases} v_p - \text{road of the class Z and lower (desgin speed)} \\ v_m - \text{road of the class G and upper (reliable speed)} \end{cases}$$

g- acceleration due to gravity $g \approx 9{,}81 \ \mathrm{m/s^2}$

b- wheelbase vehicle (passenger car 1.5-1.8 m, lorry 2.0-2.4 m)

h- height of the center of gravity of the vehicle (passenger car 0.9-1.2 m, lorry 1.5-1.6 m)

i_0 - the transverse slope of the road on the curve [-]

$$R_{\min} = \begin{cases} \dfrac{v_p^{\,2}}{g \cdot \left(\dfrac{b}{2h} - i_0\right)} [m] & \text{– slope of the trafficway in the shape of a canopy} \\ \dfrac{v_p^{\,2}}{g \cdot \left(\dfrac{b}{2h} + i_0\right)} [m] & \text{– one-side slope of the trafficway} \end{cases}$$

$$R_{\min}^{(2)} = \frac{16{,}67^2}{9{,}81 \cdot (\frac{1{,}50}{2 \cdot 1{,}20} + 0{,}04)} = 42{,}60m$$

	i_o [%]	$R_{min}^{(2)}$	
		+ i_o	**– i_o**
W1	4,0	**42,60**	—
W2	3,5	**42,92**	—
W3	3,0	**43,25**	—
W4	5,0	**41,97**	—

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b) slip stability condition

$$R_{\min} = \frac{v^2}{g \cdot (\varphi_R \pm i_0)} \text{ [m], where:}$$

v- speed [m/s]

$$v = \begin{cases} v_p - \text{road of the class Z and lower (desgin speed)} \\ v_m - \text{road of the class G and upper (reliable speed)} \end{cases}$$

g- acceleration due to gravity $g \approx 9{,}81\ \mathrm{m/s^2}$

φ_R - coefficient of transverse adhesion of the tire to the road

i_0 - the transverse slope of the road on the curve [-]

$$R_{\min} = \begin{cases} \dfrac{v_p^{\ 2}}{g \cdot (\varphi_R - i_0)} [m] & \text{– slope of the trafficway in the shape of a canopy} \\ \dfrac{v_p^{\ 2}}{g \cdot (\varphi_R + i_0)} [m] & \text{– one-side slope of the trafficway} \end{cases}$$

$\varphi_R = \mathbf{0{,}2}$ [-]

wet asphalt surface

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$$R_{\min}^{(3)} = \frac{16{,}67^2}{9{,}81 \cdot (0{,}20 + 0{,}04)} = 118{,}03m$$

	i_o [%]	$R_{min}^{(3)}$ [m]	
		+ i_o	**– i_o**
W1	4,0	**118,03**	—
W2	3,5	**120,54**	—
W3	3,0	**123,16**	—
W4	5,0	**113,31**	—

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c) driving comfort condition

$$R_{\min} = \frac{v^2}{g \cdot (\mu \pm i_0)} \quad [\mathrm{m}], \text{ where:}$$

v - speed [m/s]

$$v = \begin{cases} v_p - \text{road of the class Z and lower (desgin speed)} \\ v_m - \text{road of the class G and upper (reliable speed)} \end{cases}$$

g - acceleration due to gravity $g \approx 9.81\ \mathrm{m/s^2}$

μ - transverse acceleration factor [-]

i_0 - the transverse slope of the road on the curve [-]

$$R_{\min} = \begin{cases} \dfrac{{v_p}^2}{g \cdot (\mu - i_0)} [m] & \text{– slope of the trafficway in the shape of a canopy} \\ \dfrac{{v_p}^2}{g \cdot (\mu + i_0)} [m] & \text{– one-side slope of the trafficway} \end{cases}$$

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It stands out due to the driving comfort:

- $\mu = 0{,}02$ – a passenger who does not observe the road, will not distinguish driving between section of straight or curved; the driver feels no tension
- $\mu = 0{,}06$ – a passenger has a poor feel of driving along the curvilinear section; the driver feels small tension
- $\mu = 0{,}10$ – a passenger feels the driving along the curvilinear section, but does not feel discomfort; the driver clearly feels the tension
- $\mu = 0{,}17$ – driving along a curvilinear section is inconvenient for everyone

adopted **$\mu = 0{,}10$ [-]**, possibly $\mu = 0{,}12$ [-]

$$R_{\min}^{(4)} = \frac{16{,}67^2}{9{,}81 \cdot (0{,}10 + 0{,}04)} = 202{,}34m$$

	i_o [%]	$R_{\min}^{(4)}$	
		$+ i_o$	**$- i_o$**
W1	4,0	**202,34**	—
W2	3,5	**209,83**	—
W3	3,0	**217,90**	—
W4	5,0	**188,85**	—

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JUXTAPOSITION R_{min}

	R [(1)] JoL 16	$\mathbf{R}_{\mathbf{min}}$ [(2)] roll-over	$\mathbf{R}_{\mathbf{min}}$ [(3)] slip	$\mathbf{R}_{\mathbf{min}}$ [(4)] comfort	$\mathbf{i}_{\mathbf{o}}$ [%]	Adopted **R [m]**
W1	250	42,60	118,03	202,34	4,0	**250**
W2	320	42,92	120,54	209,83	3,5	**320**
W3	380	43,25	123,16	217,90	3,0	**380**
W4	200	41,97	113,31	188,85	5,0	**200**

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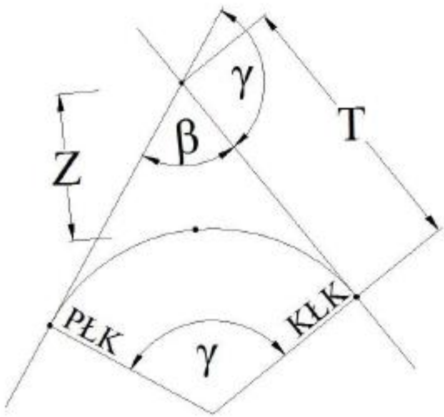
Calculation of basic elements of a horizontal curve



Signs:
PŁK – BC – begin of curve
KŁK – EC – end of curve
Z – E – external
T – tangent

Curve 1

$\gamma_1 = 51{,}6331^o$ $R_1 = 250$ m

– calculating the tangent of a curve

$$T_1 = R_1 \cdot tg\frac{\gamma_1}{2} = 250 \cdot tg\frac{51{,}6331^o}{2} = 120{,}94\ m$$

– calculating the external of a curve

$$Z_1 = \frac{R_1}{\cos\frac{\gamma_1}{2}} - R_1 = 250 \cdot \left(\frac{1}{\cos\frac{51{,}6331^o}{2}} - 1\right) = 27{,}72\ m$$

– curve length calculation

$$D_1 = R_1 \cdot \frac{\pi}{180} \cdot \gamma_1 = 250 \cdot \frac{\pi}{180} \cdot 51{,}6331^o = 225{,}29m$$

– calculation of the widening on a curve

$$\frac{40}{R} = \frac{40}{250} = 0{,}16m$$

The widening is used when its value is greater than or equal to 0.2 m

	R [m]	γ [°]	T [m]	Z [m]	D [m]	$\frac{40}{R}$	p [m]
Curve 1	250	51,6331	120,94	27,72	225,29	0,16	–
Curve 2	320	40,4684	117,95	21,05	226,02	0,13	–
Curve 3	380	34,5163	118,05	17,91	228,92	0,11	–
Curve 4	200	48,7691	90,66	19,59	170,24	0,20	0,20

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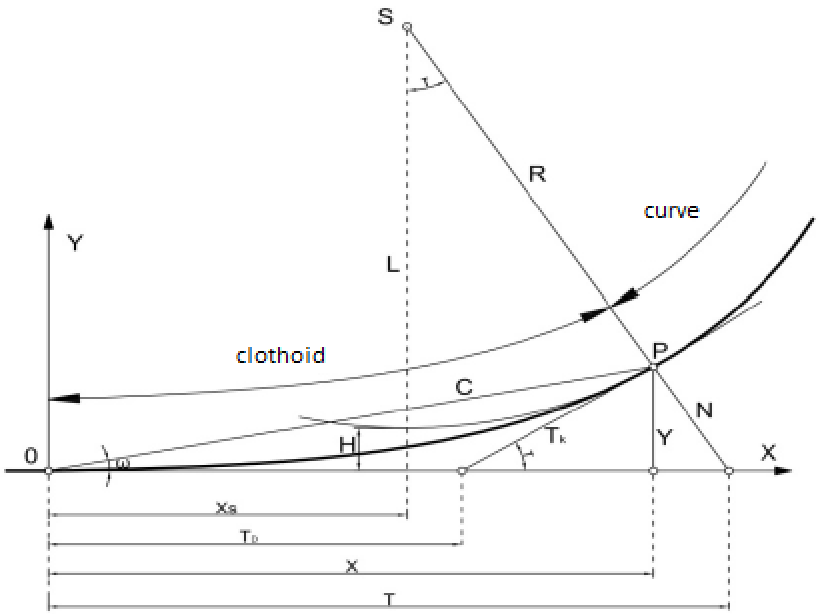
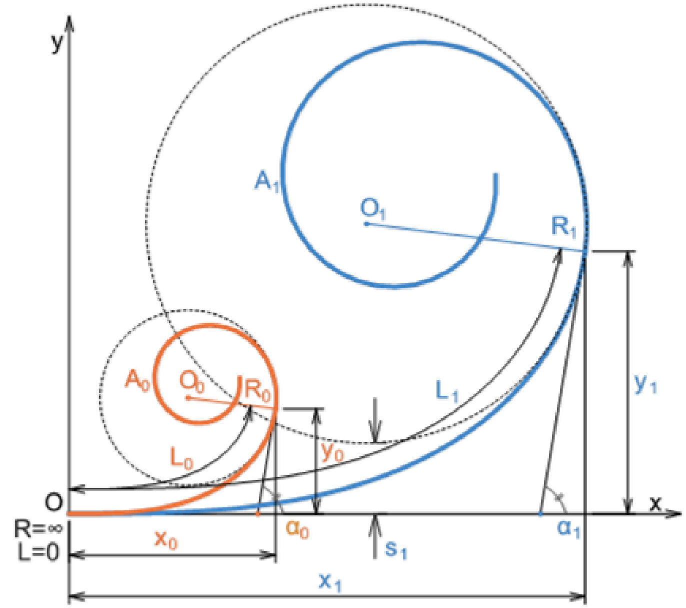
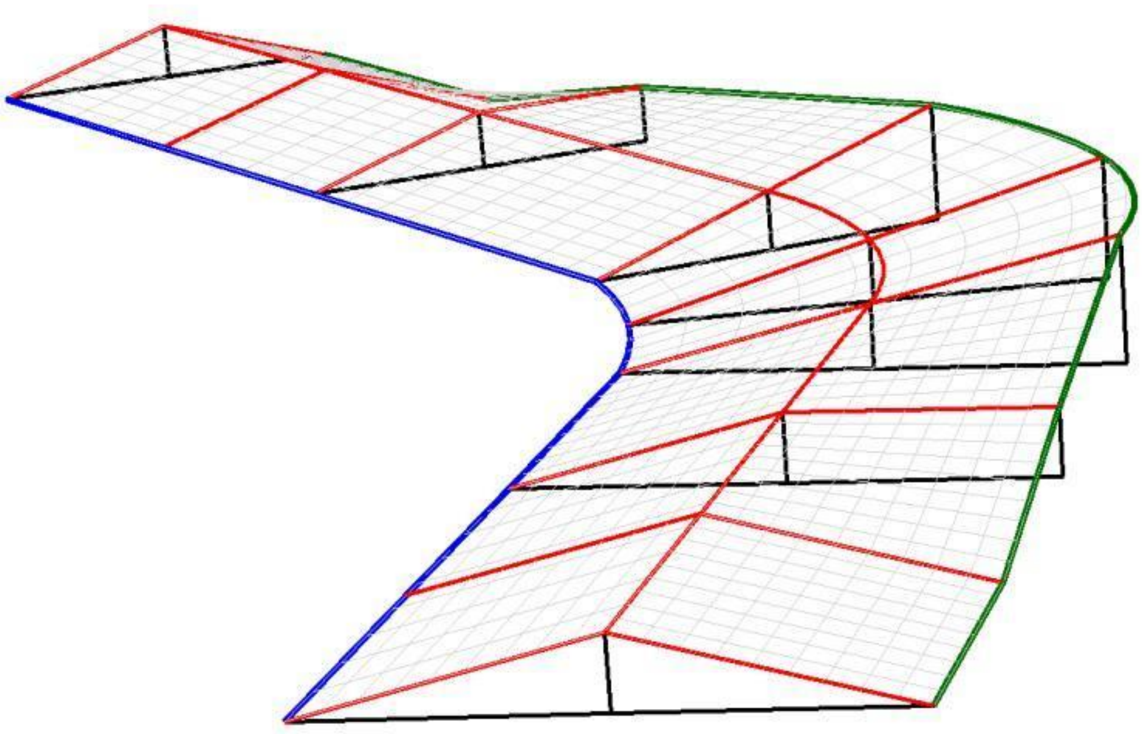
Determination of the clothoid a-parameter





Source: https://pwayblog.com/2016/07/03/the-clothoid/

The road ramp



Source: https://docplayer.pl/docs-images/64/51106521/images/11-1.jpg

Determination of the clothoid a-parameter

a) dynamics condition

$$a_{\min} = \sqrt{\frac{v^3}{k}}[m],$$ where:

a - clothoid a-parameter [m]

v - speed [m/s]

$v = v_p$ for all road classes $\Rightarrow v = v_p = 60\frac{km}{h} \Rightarrow v = 16{,}67\frac{m}{s}$

$k -$ permissible increase centripetal acceleration $\Rightarrow V_p = 60\frac{km}{h} \Rightarrow k = 0{,}7\frac{m}{s^3}$

$$a_{\min} = \sqrt{\frac{16{,}67^3}{0{,}7}} = 81{,}35m$$

b) aesthetics condition

$$a_{min} = \frac{1}{3} R[m]$$

$a \quad = R[m]$, where:

R- radius of the horizontal curve [m]

$$a_{1min} = \frac{1}{3} \cdot 200 = 66{,}67m$$

$$a_{1max} = 200m$$

	R [m]	a_{min} [m]	a_{max} [m]
curve 1	250	**83,33**	**250**
curve 2	320	**106,67**	**320**
curve 3	380	**126,67**	**380**
curve 4	200	**66,67**	**200**

c) construction of a road ramp condition

$$a_{\min} = \sqrt{\frac{R \cdot B}{2} \cdot \frac{i_n + i_o}{i_{d\max}}} \text{ [m]},$$ where:

R – radius of the horizontal curve [m]

B – roadway width [m]; traffic line width outside the built-up area for a Z-class road is 2.75-3.00 m; adopted 6,00 m

i_o – the transverse slope of the roadway on a curvilinear section [-]

i_n – the transverse slope of the roadway on a straight section

i_d – additional slope of the roadway on the road ramp [-]

$$i_{d\min} \le i_d \le i_{d\max}$$

$$i_{d\min} = 0{,}1 \cdot \frac{B}{2} = 0{,}1 \cdot \frac{6{,}00}{2} = 0{,}3\%$$

$i_{d\max} = 1{,}6\%$ for design speed $v_P = 60 \frac{km}{h}$

$0{,}003 \le i_d \le 0{,}016$ adopted $i_d = 0{,}016$

$$a_{1min} = \sqrt{\frac{250 \cdot 6{,}00}{2} \cdot \frac{0{,}02 + 0{,}04}{0{,}016}} = 53{,}03m$$

	R [m]	B [m]	i_n [%]	i_o [%]	i_d [%]	a_{min} [m]
curve 1	250	6,00	2,0	4,0	1,6	**53,03**
curve 2	320	6,00	2,0	3,5	1,6	**57,45**
curve 3	380	6,00	2,0	3,0	1,6	**59,69**
curve 4	200	6,40	2,0	5,0	1,6	**52,92**

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d) widening of the roadway condition

calculated for the horizontal curves which have widening

$$a_{\min} = 1{,}86 \cdot \sqrt[4]{R^3 \cdot p_c} \quad [\mathrm{m}],$$ where:

R – radius of the horizontal curve [m]
p_c – complete widening of the roadway on the curve [m]

$$a_{4\min} = 1{,}86 \cdot \sqrt[4]{200^3 \cdot 0{,}4} = 78{,}67\ m$$

e) geometric condition

$$a_{\max} = R \cdot \sqrt{\gamma} \quad [\mathrm{m}],$$ where:

R – radius of the horizontal curve [m]
γ – deflection angle of the horizontal curve [rad]

$$a_{1\max} = 250 \cdot \sqrt{0{,}901167} = 237{,}32\ m$$

	R [m]	γ [rad]	a_{max} [m]
curve 1	250	0,901167	**237,32**
curve 2	320	0,706306	**268,93**
curve 3	380	0,602422	**294,94**
curve 4	200	0,851181	**184,52**

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f) horizontal curve offset condition

recommended condition

for $H_{min} = 0{,}50m \Rightarrow a_{min} = 1{,}863 \cdot R^{\frac{3}{4}}$ [m],

for $H_{max} = 2{,}50m \Rightarrow a_{max} = 2{,}783 \cdot R^{\frac{3}{4}}$ [m], where:

H – horizntal curve offset [m]

$$a_{1min} = 1{,}863 \cdot 250^{\frac{3}{4}} = 117{,}00m$$

$$a_{1max} = 2{,}783 \cdot 250^{\frac{3}{4}} = 174{,}97m$$

	R [m]	a_{min} [m]	a_{max} [m]
curve 1	250	**117,00**	**174,97**
curve 2	320	**140,80**	**210,56**
curve 3	380	**160,17**	**239,53**
curve 4	200	**98,97**	**148,01**

g) proportionality condition

recommended condition

Signs:

L – Cl – the length of the clothoid

Ł – Cu – the length of the curve

for $L:Ł:L = 1:4:1 \Rightarrow a_{min} = R \cdot \sqrt{\frac{\gamma}{5}}$ [m],

for $L:Ł:L = 1:1:1 \Rightarrow a_{max} = R \cdot \sqrt{\frac{\gamma}{2}}$ [m], gdzie:

R- radius of the horizontal curve [m]

γ- deflection angle [rad]

$$a_{1min} = 250 \cdot \sqrt{\frac{0,901167}{5}} = 106,13m$$

$$a_{1max} = 250 \cdot \sqrt{\frac{0,901167}{2}} = 167,81m$$

	R [m]	γ [rad]	a_{min} [m]	a_{max} [m]
curve 1	250	0,901167	**106,13**	**167,81**
curve 2	320	0,706306	**120,27**	**190,17**
curve 3	380	0,602422	**131,90**	**208,55**
curve 4	200	0,851181	**82,52**	**130,47**

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JUXTAPOSITION OF A-PARAMETER VALUES

	R [m]	a_{min} [m] dyn	a_{min} [m] asth	a_{min} [m] const	a_{min} [m] widening	a_{min} [m] offset	a_{min} [m] prop
curve 1	250	81,33	83,33	53,03	–	117,00	106,13
curve 2	320	81,33	106,67	57,45	–	140,80	120,27
curve 3	380	81,33	126,67	59,69	–	160,17	131,90
curve 4	200	81,33	66,67	52,92	78,67	98,97	82,52

	R [m]	a_{max} [m] asth	a_{max} [m] geom	a_{max} [m] offset	a_{max} [m] prop
curve 1	250	250	237,32	174,97	167,81
curve 2	320	320	268,93	210,56	190,17
curve 3	380	380	294,94	239,53	208,55
curve 4	200	200	184,52	148,01	130,47

ADOPTION OF A-PARAMETER VALUE

$$a_{min}^{(max)} \leq a \leq a_{max}^{(min)}$$

	R [m]	$a_{min}^{(max)}$ [m]	$a_{max}^{(min)}$ [m]	a [m]
curve 1	250	117,00	167,81	**136,93**
curve 2	320	140,80	190,17	**154,92**
curve 3	380	160,17	208,55	**169,94**
curve 4	200	98,97	130,47	**109,55**

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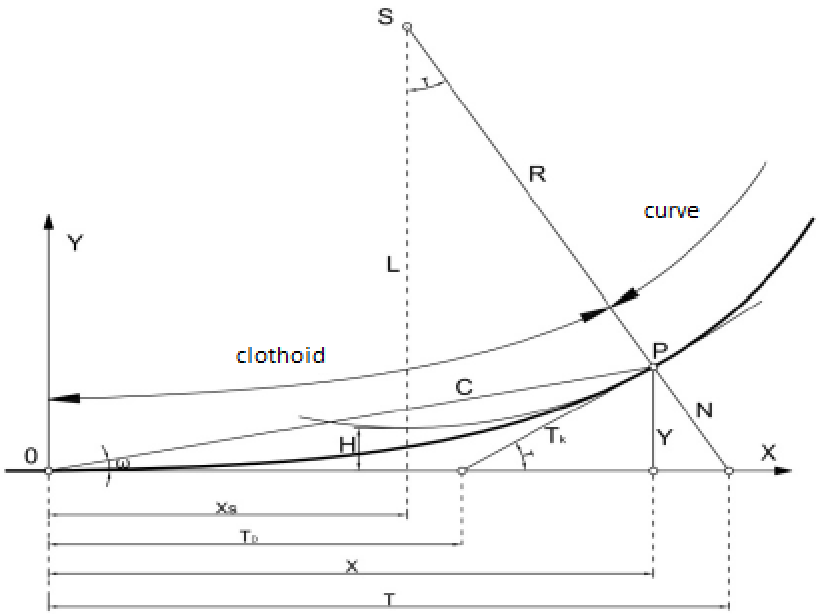
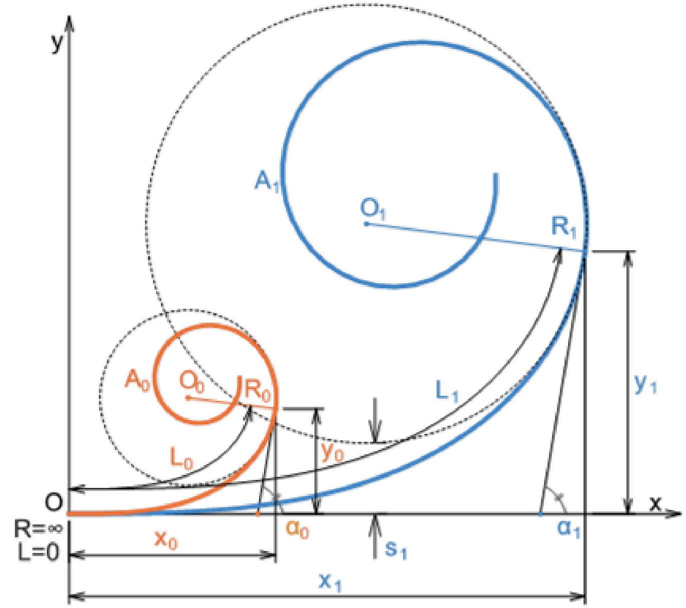
Determination of characteristic values of a colloidal transition curve





Source: https://pwayblog.com/2016/07/03/the-clothoid/

a) clothoid length

$$L = \frac{a^2}{R}$$ [m], where:

L- length of the clothoid [m]
a- clothoid a-parameter [m]
R- radius of horizntal curve [m]

$$L = \frac{136,93^2}{250} = 75,00\,m$$

	R [m]	a [m]	**L [m]**
curve 1	250	136,93	**75,00**
curve 2	320	154,92	**75,00**
curve 3	380	169,94	**76,00**
curve 4	200	109,55	**60,00**

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b) tangent angle at the end of the transition curve

$$\tau = \frac{L}{2 \cdot R} \quad [m]$$

$\tau_{min} = 3^\circ \leq \tau \leq \tau_{max} = 30^\circ$, where:

τ- tangent angle at the end of the transition curve [rad]
L- length of the clothoid [m]
R- radius of the horizontal curve [m]

$$\tau = \frac{75,00}{2 \cdot 250} = 0,150000 \quad rad$$

	R [m]	**L** [m]	τ [rad]	τ [°]	
curve 1	250	75,00	0,150000	8°35'40"	**8,5944**
curve 2	320	75,00	0,117188	6°42'52"	**6,7143**
curve 3	380	76,00	0,100000	5°43'46"	**5,7296**
curve 4	200	60,00	0,150000	8°35'40"	**8,5944**

For all transition curve the condition $\tau_{min} = 3^\circ \leq \tau \leq \tau_{max} = 30^\circ$ **is fulfilled**

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c) coordinates of the end of the transition curve

$$X = L - \frac{L^5}{40a^4} + \frac{L^9}{3456a^8} - (\ldots)[m] \qquad Y = \frac{L^3}{6a^2} - \frac{L^7}{336a^6} + \frac{L^{11}}{42240a^{10}} - (\ldots)[m],$$ where:

X- coordinates of the end of the transition curve [m]

Y- coordinates of the end of the transition curve [m]

$$X = 75{,}00 - \frac{75{,}00^5}{40 \cdot 136{,}93^4} + (\ldots) = 74{,}83m \qquad Y = \frac{75{,}00^3}{6 \cdot 136{,}93^2} - \frac{75{,}00^7}{336 \cdot 136{,}93^6} + (\ldots) = 3{,}74m$$

	L [m]	**a** [m]	**X** [m]	**Y** [m]
curve 1	75,00	136,93	**74,83**	**3,74**
curve 2	75,00	154,92	**74,90**	**2,93**
curve 3	76,00	169,94	**75,92**	**2,53**
curve 4	60,00	109,55	**59,87**	**3,00**

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d) coordinates of the center of horizontal curve

$X_s = X - (R \cdot \sin\tau)$ [m] $Y_s = Y + (R \cdot \cos\tau)$ [m], where:

X_S- coordinates of the center of reduced horizontal curve [m]
Y_S- coordinates of the center of reduced horizontal curve [m]
τ- deflection angle of tangent on the end of the transition curve [rad]

$X_s = 74{,}83 - (250 \cdot \sin 0{,}150000) = 37{,}47m$ $Y_s = 3{,}74 + (250 \cdot \cos 0{,}150000) = 250{,}94m$

	R [m]	**X** [m]	**Y** [m]	τ [rad]	**X_s** [m]	**Y_s** [m]
curve 1	250	74,83	3,74	0,150000	**37,47**	**250,94**
curve 2	320	74,90	2,93	0,117188	**37,48**	**320,73**
curve 3	380	75,92	2,53	0,100000	**37,99**	**380,63**
curve 4	200	59,87	3,00	0,150000	**29,98**	**200,75**

e) retraction of the horizontal curve

$$H = Y - R \cdot (1 - \cos\tau) \ [m]$$

$$H_{min} = 0{,}5m \le H \le H_{max} = 2{,}5m$$, where:

H- retraction of the horizontal curve [m]

τ- deflection angle of the tangent on the end of the horizontal curve [rad]

$$H = Y - R \cdot (1 - \cos\tau) = 3{,}74 - 250 \cdot (1 - \cos 0{,}150000) = 0{,}94m$$

	R [m]	**Y** [m]	**τ** [rad]	**H** [m]
curve 1	250	3,74	0,150000	**0,94**
curve 2	320	2,93	0,117188	**0,73**
curve 3	380	2,53	0,100000	**0,63**
curve 4	200	3,00	0,150000	**0,75**

For all transition curve the condition $H_{min} = 0{,}5m \le H \le H_{max} = 2{,}5m$ **is fulfilled**

f) external

$$N = \frac{Y}{\cos \tau} \text{ [m], where:}$$

N- external of the transition curve [m]

τ- deflection angle of the tangent on the end of the horizontal curve [rad]

$$N = \frac{3,74}{\cos\ 0,150000} = 3,79\ m$$

	Y [m]	τ [rad]	N [m]
curve 1	3,74	0,150000	**3,79**
curve 2	2,93	0,117188	**2,95**
curve 3	2,53	0,100000	**2,54**
curve 4	3,00	0,150000	**3,03**

g) short tangent

$$T_K = \frac{Y}{\sin \tau} \text{ [m], where:}$$

T_K- short tangent [m]
τ- deflection angle of the tangent on the end of the horizontal curve [rad]

$$T_K = \frac{3,74}{\sin 0,150000} = 25,05 \; m$$

	Y [m]	τ [rad]	T_K [m]
curve 1	3,74	0,150000	**25,05**
curve 2	2,93	0,117188	**25,03**
curve 3	2,53	0,100000	**25,36**
curve 4	3,00	0,150000	**20,04**

h) long tangent

$$T_D = X - Y \cdot ctg\ \tau \quad [m],$$ where:

T_D- long tangent [m]
τ- deflection angle of the tangent on the end of the horizontal curve [rad]

$$T_D = 74,83 - 3,74 \cdot ctg\ 0,150000 = 50,06\ m$$

	X [m]	Y [m]	τ [rad]	T_D [m]
curve 1	74,83	3,74	0,150000	**50,06**
curve 2	74,90	2,93	0,117188	**50,04**
curve 3	75,92	2,53	0,100000	**50,69**
curve 4	59,87	3,00	0,150000	**40,05**

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i) T_s section

$$T_S = (R + H) \cdot tg\frac{\gamma}{2} \ \ [m], \text{ where:}$$

T_S- length of the section T_S [m]

γ- deflection angle of the horizontal curve [rad]

$$T_S = (250 + 0{,}94) \cdot tg\frac{0{,}901167}{2} = 121{,}40m$$

	R [m]	**H** [m]	γ [rad]	**T_s** [m]
curve 1	250	0,94	0,901167	**121,40**
curve 2	320	0,73	0,706306	**118,22**
curve 3	380	0,63	0,602422	**118,25**
curve 4	200	0,75	0,851181	**91,00**

j) integer tangent

$T_0 = X_s + T_S$ [m], where:

T_0- integer tangent [m]

$T_0 = 37{,}47 + 121{,}40 = 158{,}87$ m

	X_s [m]	T_s [m]	T_0 [m]
curve 1	37,47	121,40	**158,87**
curve 2	37,48	118,22	**155,71**
curve 3	37,99	118,25	**156,24**
curve 4	29,98	91,00	**120,98**

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k) central angle of the reduced horizontal curve

$\alpha = \gamma - 2 \cdot \tau$ [m], where:

γ- deflection angle of the horizontal curve [rad]
τ- deflection angle of the tangent on the end of the horizontal curve [rad]
α- central angle of the reduced horizontal curve [rad]

$\alpha = 0{,}901167 - 2 \cdot 0{,}150000 = 0{,}601167$ rad

	γ [rad]	τ [rad]	α [rad]	α [°]	
curve 1	0,901167	0,150000	0,601167	34°26'40"	**34,4443**
curve 2	0,706306	0,117188	0,471931	27°02'23"	**27,0397**
curve 3	0,602422	0,100000	0,402422	23°03'26"	**23,0571**
curve 4	0,851181	0,150000	0,551181	31°34'49"	**31,5803**

l) length of the reduced horizontal curve

$Ł = R \cdot \alpha$ [m], where:

α- central angle of the reduced horizontal curve [rad]

Ł- length of the reduced horizontal curve [m]

$Ł = 250 \cdot 0,601167 = 150,29$ m

	R [m]	α [rad]	Ł [m]
curve 1	250	0,601167	**150,29**
curve 2	320	0,471931	**151,02**
curve 3	380	0,402422	**152,92**
curve 4	200	0,551181	**110,24**

Coordinates for staking out the transition curve

Intermediate points for the transition curves in local coordinate systems

$$x(l) = l - \frac{l^5}{40a^4} + \frac{l^9}{3456a^8} - (\ldots)[m] \qquad y(l) = \frac{l^3}{6a^2} - \frac{l^7}{336a^6} + \frac{l^{11}}{42240a^{10}} - (\ldots)[m],$$ where:

x(l) - local coordinate of the intermediate point of the transition curve [m]
y(l) - local coordinate of the intermediate point of the transition curve [m]

where: $0 \le l \le L$

$$x(l) = 50{,}00 - \frac{50{,}00^5}{40 \cdot 136{,}93^4} + (\ldots) = 49{,}98m \qquad y(l) = \frac{50{,}00^3}{6 \cdot 136{,}93^2} - \frac{50{,}00^7}{336 \cdot 136{,}93^6} + (\ldots) = 1{,}11m$$

Transformation of the coordinate from the local system to the global system

$$\begin{cases} X = x \cdot \cos\alpha - y \cdot \sin\alpha + a \\ Y = x \cdot \sin\alpha + y \cdot \cos\alpha + b \end{cases}$$

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Mileage of the horizontal alignment

		0+000,00	PA=PPT
$+AW_1$	+	619,90	
		0+619,90	W_1
$-T_{01}$	−	158,87	
		0+461,04	PKP_1
$+L_1$	+	75,00	
		0+536,04	$KKP_1=PŁK_1$
$+Ł_1/2$	+	75,15	
		0+611,18	$ŚŁK_1$
$+Ł_1/2$	+	75,15	
		0+686,33	$KŁK_1=KKP_1$
$+L_1$	+	75,00	
		0+761,33	PKP_1
$-T_{01}$	−	158,87	
		0+602,45	W_1^*
$+W_1W_2$	+	454,56	
		1+057,01	W_2
$-T_{02}$	−	155,71	
		0+901,31	PKP_2
$+L_2$	+	75,00	
		0+976,31	$KKP_2=PŁK_2$
$+Ł_2/2$	+	75,51	
		1+051,82	$ŚŁK_2$
$+Ł_2/2$	+	75,51	
		1+127,33	$KŁK_2=KKP_2$
$+L_2$	+	75,00	
		1+202,33	PKP_2
$-T_{02}$	−	155,71	
		1+046,62	W_2^*
$+W_2W_3$	+	725,36	
		1+771,98	W_3
$-T_{03}$	−	156,24	
		1+615,75	PKP_3
$+L_3$	+	76,00	
		1+691,75	$KKP_3=PŁK_3$
$+Ł_3/2$	+	76,46	
		1+768,21	$ŚŁK_3$
$+Ł_3/2$	+	76,46	
		1+844,67	$KŁK_3=KKP_3$

$$W_1-W_1^* = 2\cdot T_0-2\cdot L-Ł$$
$$619{,}90-602{,}45 = 2\cdot 158{,}87-2\cdot 75{,}00-150{,}29$$
$$17{,}45 = 17{,}45$$

$$W_2-W_2^* = 2\cdot T_0-2\cdot L-Ł$$
$$1057{,}01-1046{,}62 = 2\cdot 155{,}71-2\cdot 75{,}00-151{,}02$$
$$10{,}40 = 10{,}40$$

$$W_3-W_3^* = 2\cdot T_0-2\cdot L-Ł$$
$$1771{,}98-1764{,}43 = 2\cdot 156{,}24-2\cdot 76{,}000-152{,}92$$
$$7{,}55 = 7{,}55$$

Signs:

PA – PPT – BDR – begin of the design road

W – V – vertex point

PŁK – BC – begin of the curve

KŁK – EC – end of the curve

PKP – BTC – begin of the transition curve

KKP – ETC – end of the transition curve

ŚŁK – CC – center of the curve

PB – KPT – EDR – end of the design road

L – the length of the clothoid

Ł – L’ – the length of the reducted curve

T_o – the length of the integer tangent

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$+L_3$	+	76,00	
		1+920,67	PKP_3
$-T_{03}$	–	156,24	
		1+764,43	W_3*
$+W_3W_4$	+	943,65	
		2+708,08	W_4
$-T_{04}$	–	120,98	
		2+587,10	PKP_4
$+L_4$	+	60,00	
		2+647,10	KKP_4=$PŁK_4$
$+Ł_4/2$	+	55,12	
		2+702,22	$ŚŁK_4$
$+Ł_4/2$	+	55,12	
		2+757,34	$KŁK_4$=KKP_4
$+L_4$	+	60,00	
		2+817,34	PKP_4
$-T_{04}$	–	120,98	
		2+696,36	W_4*
$+W_4B$	+	285,94	
		2+982,30	B

$$W_4 - W_4^* = 2\cdot T_0 - 2\cdot L - Ł$$
$$2708{,}08-2696{,}36 = 2\cdot 120{,}98-2\cdot 60{,}000-110{,}24$$
$$11{,}72 = 11{,}72$$

Verification:

$AW_1 - (W_1 - W_1^*) + W_1W_2 - (W_2 - W_2^*) + W_2W_3 - (W_3 - W_3^*) + W_3W_4 - (W_4 - W_4^*) + W_4B =$ KT

619,90 – (619,90–602,45) + 454,56 – (1057,01–1046,62) + 725,36

– (1771,98–1764,43) + 943,65 – (2708,08–2696,36) + 285,94 = 2982,30

2982,30 = 2982,30

Literature:

http://onlinemanuals.txdot.gov/txdotmanuals/rdw/horizontal_alignment.htm

https://engineering.purdue.edu/~ce361/JFRICKER/HW/HC_02fHW6.pdf

THANK YOU FOR YOUR ATTENTION

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