Fundamentals of Road Construction

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Uczelnia zintegrowana na przyszłość
POWR.03.05.00-00-Z041/17

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Lecture 4

The subject of the lecture:

Vertical alignment

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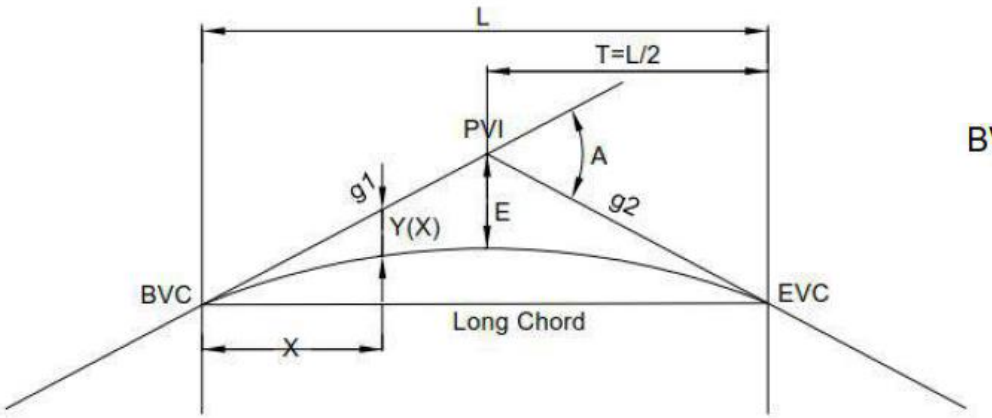
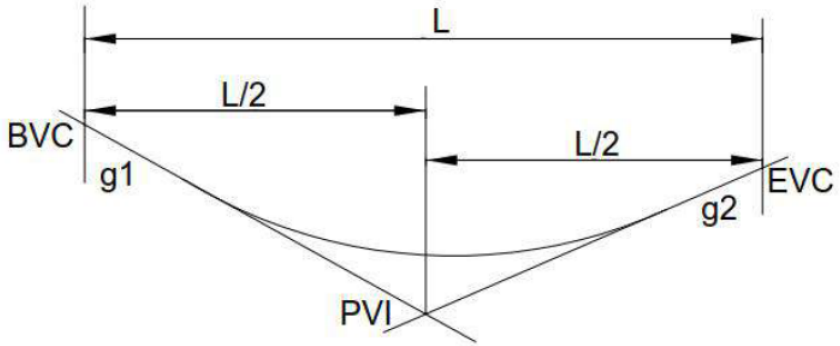
Vertical alignment is the combination of circular vertical curves and tangent (straight) sections of a particular slope designed to achieve this objective. The design of vertical alignment is concerned with gradients, crest and sag curves. A crest curve is a convex vertical curve. A sag curve is a concave vertical curve.



a crest curve



a sag curve

Source: Low Volume Roads Manual, Volume 2: Geometric Design and Road Safety, Roads Authority Malawi, 2020

Terrain vertical following-coordinates:

No.	Mileage	Vertical coordinates [m]	Description
1.	0+000,00	647,50	PA , contour line
2.	0+058,20	645,00	contour line
3.	0+100,00	643,00	Hm - hectometer
4.	0+120,74	642,50	contour line
5.	0+173,98	640,00	contour line
...	...	...	...
17.	0+417,29	635,00	contour line
18.	0+441,13	637,50	contour line
19.	0+500,00	638,95	Hm - hectometer
20.	0+500,89	639,00	PKP_1
21.	0+581,53	639,50	$KKP_1 = PŁK_1$
22.	0+600,00	639,80	Hm - hectometer
23.	0+639,88	639,70	$ŚŁK_1$
24.	0+698,23	638,80	$KKP_1 = KŁK_1$
...	...	...	...
90.	2+999,34	617,50	PB , contour line

Signs:

PA – PPT – BDR – begin of the design road

PŁK – BC – begin of the curve

KŁK – EC – end of the curve

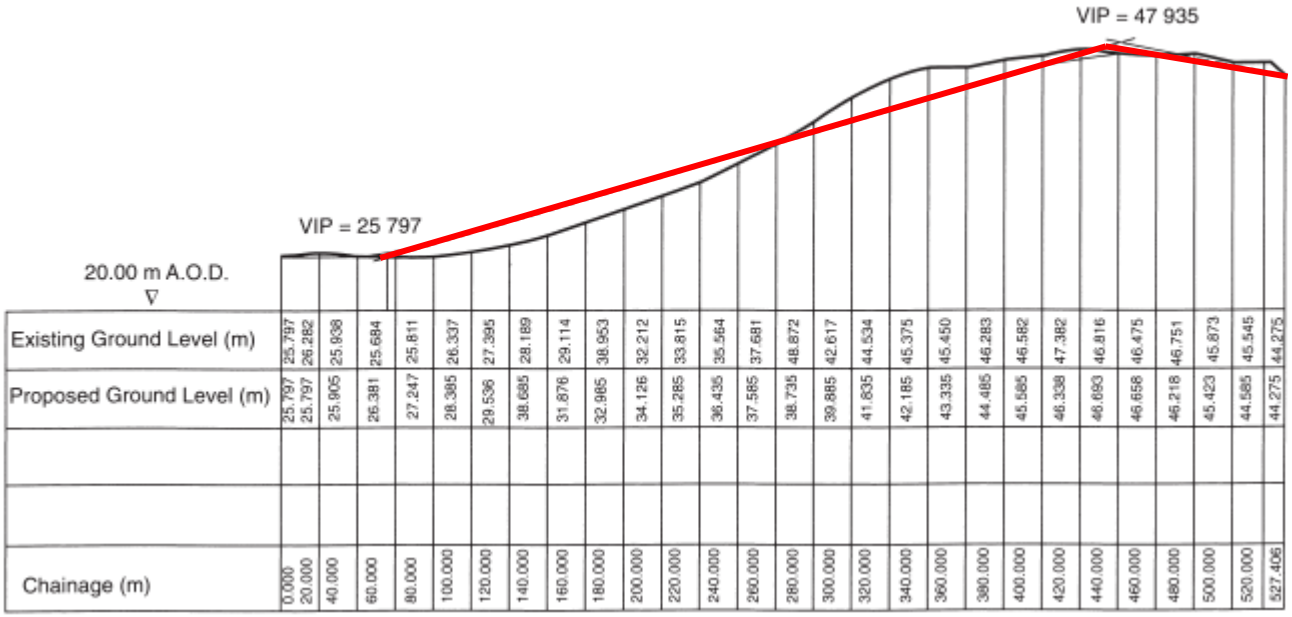
PKP – BTC – begin of the transition curve

KKP – ETC – end of the transition curve

ŚŁK – CC – center of the curve

PB – KPT – EDR – end of the design road

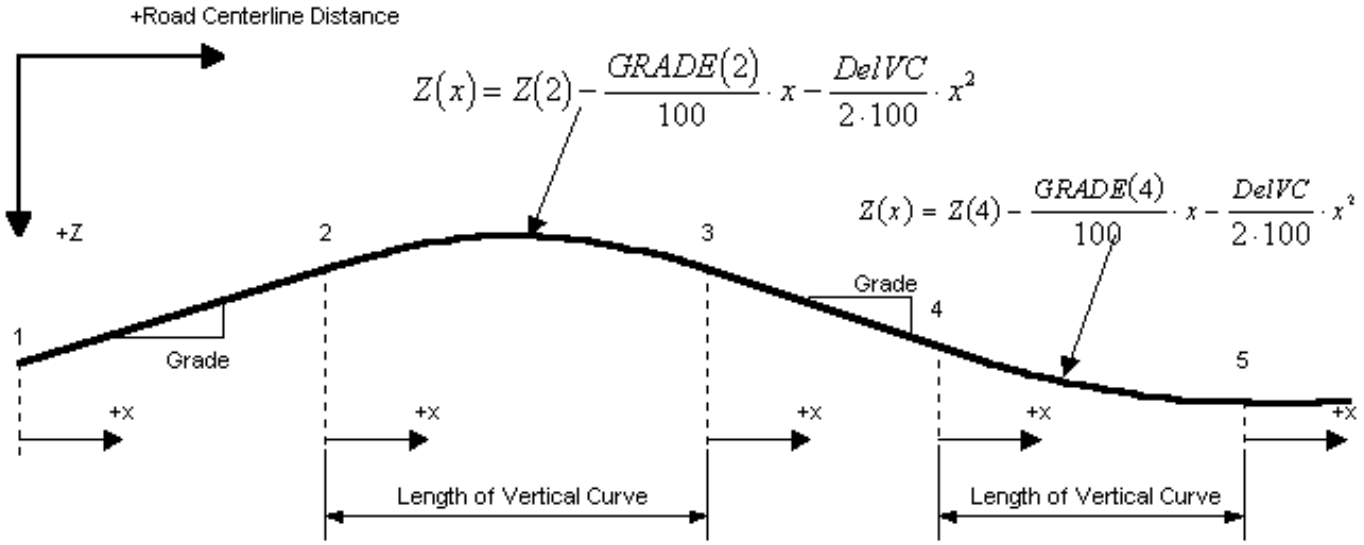
Longitudinal section of a raod and terrain profile:



Existing Ground Level (m)	25.797	26.282	25.938	25.684	25.811	26.337	27.395	28.189	29.114	30.953	32.212	33.815	35.564	37.681	38.872	42.617	44.534	45.375	45.450	46.283	46.582	47.382	46.816	46.475	46.751	45.873	45.545	44.275	
Proposed Ground Level (m)	25.797	25.797	25.905	26.381	27.247	28.385	29.536	30.685	31.876	32.985	34.126	35.285	36.435	37.585	38.735	39.885	41.835	42.185	43.335	44.485	45.585	46.338	46.693	46.658	46.218	45.423	44.585	44.275		
Chainage (m)	0.000	20.000	40.000	60.000	80.000	100.000	120.000	140.000	160.000	180.000	200.000	220.000	240.000	260.000	280.000	300.000	320.000	340.000	360.000	380.000	400.000	420.000	440.000	460.000	480.000	500.000	520.000	527.406		

red line – a grade line

Geometric elements of the grade line:



Źródło: http://web.mit.edu/16.400/www/auto_sim/Help/SDLEventVC.htm

Determination of the parameters of the grade line on sections with a constant slope:

a) ordinate of the points of bend of the grade line

Point	Mileage	Ordinate of breakdown [m a.s.l.]
A	0+000,00	646,50
Z1	0+230,00	638,00
Z2	0+639,88	640,50
Z3	1+330,00	624,60
Z4	2+380,00	612,45
Z5	2+718,36	611,00
B	2+999,34	616,50

b) slope of the grade line on sections with a constant slope

$$i = \frac{H_k - H_p}{L} \cdot 100 \ [\%],$$ where:

i – slope of the grade line on sections with a constant slope [%], between points of breakdown of the grade line niwelety),
H_p – ordinate of a beginning of section of grade line with constant slope [m]
H_k – ordinate of a beginning of section of grade line with constant slope [m]
L – length of section of grade line with constant slope [m]

$$i = \frac{638{,}00 - 646{,}50}{230{,}00} \cdot 100 = -3{,}696\%$$

c) the angle of bend of the grade line

$$\alpha = |i_n - i_{n+1}| \ [\%], \text{ where:}$$

α – the angle of refraction of the grade line [%]
i_n – the slope of the grade line before bend [%]
i_{n+1} – the slope of the grade line after bend [%]

$$\alpha = |i_2 - i_3| = |0{,}610 - (-2{,}304)| = 2{,}914\%$$

e) mean slope of the grade line for the bend

$$i_{śr} = \frac{i_n + i_{n+1}}{2} \ [\%], \text{ where:}$$

$i_{śr}$ – mean slope of the grade line for the bend [%]
i_n – slope of the grade line before the bend [%]
i_{n+1} – slope of the grade line after the bend [%]

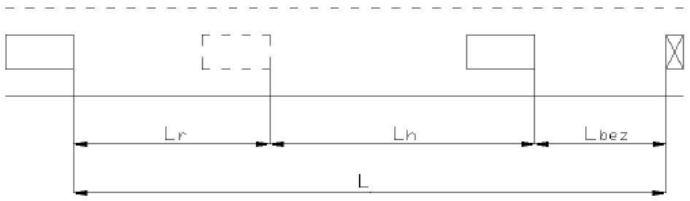
$$i_{śr} = \frac{i_2 + i_3}{2} = \frac{0{,}610 + (-2{,}304)}{2} = -0{,}847\%$$

The list of parameters of the bends of the grade line

Bend number	Mileage	Scheme	*i* before bend [%]	*i* after bend [%]	α [%]	$i_{śr}$ [%]
Z1 sag	0+230,00		-3,696	0,610	**4,306**	-1,543
Z2 crest	0+639,88		0,610	-2,304	**2,914**	-0,847
Z3 sag	1+330,00		-2,304	-1,157	1,147	-1,731
Z4 sag	2+380,00		-1,157	-0,429	0,729	-0,793
Z5 sag	2+718,36		-0,429	1,957	2,386	0,764

short Z = B - bend

Determination of the distance of the required stopping visibility:



$$L = L_r + L_h + L_{bez}$$

$$L_Z \geq L = v \cdot t + \frac{v^2}{2 \cdot g \cdot (0{,}95 \cdot \varphi + f - |i_{śr}|)} + 10 \ [\text{m}], \quad \text{where:}$$

L_Z – required distance of the stopping visibility [m]
L – length of the vehicle stopping distance [m]
v – speed [m/s]

$$v = \begin{cases} v_p + 10 - \text{for road class Z and roads of lower classes} \\ v_m - \text{for road class G and roads of higher classes} \end{cases} \Rightarrow v = v_P + 10 = 70\frac{km}{h} \Rightarrow v = 19{,}44\frac{m}{s}$$

t – driver reaction time [s], adopted ***t = 1 s***
g – acceleration due to gravity [m/s²], g = 9,81 m/s²
φ – a longitudinal coefficient of adhesion of the tire to the road surface
Coefficient was adopted for the wet asphalt surface : **φ = 0,35 [-]**
f – rolling friction coefficient for asphalt pavement in average condition
adopted : **f = 0,018 [-]**
$i_{śr}$ – mean slope of the grade line [-]

$$L_Z = 19{,}44 \cdot 1 + \frac{19{,}44^2}{2 \cdot 9{,}81 \cdot (0{,}95 \cdot 0{,}35 + 0{,}018 - |0{,}01543|)} + 10 = 163{,}47m$$

List of the distance of the required stopping visibility L_Z

Bend number	Mileage	Scheme	$i_{śr}$ [–]	L [m] by formula	L_{Zmin} [m] by JoL16	adopted L_Z [m]
Z1 sag	0+230,00		-0,01543	86,96	90,00	90,00
Z2 crest	0+639,88		-0,00847	85,79	90,00	90,00
Z3 sag	1+330,00		-0,01731	87,28	90,00	90,00
Z4 sag	2+380,00		-0,00793	85,70	90,00	90,00
Z5 sag	2+718,36		0,00764	85,65	90,00	90,00

Determination of the radius of vertical curves :

a) minimum radius due to the visibility for the crest curve (curve no 2)

- requirement 1

$$L_Z \le \frac{a}{\alpha} \quad \rightarrow \quad R_{min} \text{ does not specify}$$

$$\frac{a}{\alpha} = \frac{1{,}00}{0{,}02914} = 34{,}32\ m$$

$$L_Z = 90{,}00\ \text{m} \ge \frac{a}{\alpha} = 34{,}32\ \text{m}, \text{ must be calculated } R_{min}$$

- requirement 2

$$\frac{a}{\alpha} < L_Z \le \frac{2a}{\alpha} \quad \rightarrow \quad R_{min} = \frac{2}{\alpha} \cdot \left(L_Z - \frac{a}{\alpha} \right) [m]$$

$$\frac{a}{\alpha} = 34{,}32\ m \qquad \frac{2a}{\alpha} = \frac{2 \cdot 1{,}00}{0{,}02914} = 68{,}64\ m$$

$L_Z = 90{,}00$ m is not within the specified range

- requirement 3

$$L_Z > \frac{2a}{\alpha} \quad \rightarrow \quad R_{\min} = \frac{L_Z^2}{2a}[m], \text{ where:}$$

R_{min} – minimum radius of vertical curve [m]
L_Z – required distance of the stopping visibility [m]
a – localization of the observation point [m] **a = 1,00m**,
α – the angle of bend of the grade line [rad]

$$L_Z = 90{,}00 \text{ m} \geq \frac{2a}{\alpha} = 68{,}64\ m$$

$$R_{\min} = \frac{90{,}00^2}{2 \cdot 1{,}00} = 4050{,}00 m$$

Bend no.	Milage	L_Z [m]	$\frac{a}{\alpha}$	$\frac{2a}{\alpha}$	Formula	R_{min} [m]
Z2 crest	0+639,88	90,00	34,32	68,64	$R_{\min} = \frac{L_Z^2}{2a}[m]$	4050,00

b) minimum radius due to the visibility for the sag curve (curve no 1, 3, 4, 5)

- requirement 1

$$\alpha \le 2\psi \;\rightarrow\; R_{min} \text{ does not specify}$$
$$\alpha = 0{,}04306 \text{ rad} \ge 2\psi = 0{,}0348 \text{ rad, must be calculated } R_{min}$$

- requirement 2

$$\frac{h}{\alpha-\psi} < L_Z \le \frac{2h}{\alpha-2\psi} \;\rightarrow\; R_{min} = \frac{2}{\alpha}\cdot\left(L_Z - \frac{h+L_Z\cdot\psi}{\alpha}\right)[m]$$

$$\frac{h}{\alpha-\psi} = 29{,}23 \qquad \frac{2h}{\alpha-2\psi} = \frac{2\cdot 0{,}75}{0{,}04306-2\cdot 0{,}0174} = 181{,}69$$

$L_Z = 90{,}00$ m is within the specified range

$$R_{min} = \frac{2}{\alpha}\cdot\left(L_Z - \frac{h+L_Z\cdot\psi}{\alpha}\right)[m]\text{, where:}$$

R_{min} – radius of the vertical curve [m]
α – the angle of bend of the grade line [%]
L_Z – required distance of the stopping visibility [m]
ψ – the elevation angle of the useful light beam from the headlight [rad], adopted: $\psi = 0{,}0174$ rad
h – height of the optical axis of the headlight above the road surface [m], adopted: $h = 0{,}75$ m

$$R_{min} = \frac{2}{0{,}04306}\cdot\left(90{,}00 - \frac{0{,}75+90{,}00\cdot 0{,}0174}{0{,}04306}\right) = 1681{,}97m$$

- requirement 3

$$L_Z > \frac{2h}{\alpha - 2\psi} \quad \rightarrow \quad R_{\min} = \frac{L_Z^2}{2 \cdot (h + L_Z \cdot \psi)} [m] \quad \text{, where:}$$

R_{min} – radius of the vertical curve [m]
α – the angle of bend of the grade line [%]
L_Z – required distance of the stopping visibility [m]
ψ – the elevation angle of the useful light beam from the headlight [rad], adopted: $\psi = 0{,}0174$ rad
h – height of the optical axis of the headlight above the road surface [m], adopted: $h = 0{,}75$ m

Bend no.	Mileage	α [rad]	2ψ [rad]	Calculation R_{min}
Z1 sag	0+230,00	0,04306	0,0348	must be calculated
Z3 sag	1+330,00	0,01147	0,0348	does not specify
Z4 sag	2+380,00	0,00729	0,0348	does not specify
Z5 sag	2+718,36	0,02386	0,0348	does not specify

Bend number	Mileage	L_Z [m]	α [rad]	$\frac{2h}{\alpha - 2\psi}$	Formula	R_{min} [m]
Z1 sag	0+230,00	90,00	0,04306	181,69	$R_{min} = \frac{2}{\alpha} \cdot \left(L_Z - \frac{h + L_Z \cdot \psi}{\alpha} \right) [m]$	1681,97

c) minimum radius due to the dynamics

$$R_{min} = 0{,}154 \cdot v^2 \text{ [m], where:}$$

R_{min} – minimum radius of the vertical curve [m]
v – speed [km/h]

$$v = \begin{cases} v_p - \text{for road class Z and roads of lower classes} \\ v_m - \text{for road class G and roads of higher classes} \end{cases} \Rightarrow v = v_p = 60\frac{km}{h}$$

$$R_{min} = 0{,}154 \cdot 60^2 = 554{,}40 \text{ m}$$

d) minimum radius due to the aesthetics

$$R_{min} = \frac{100 \cdot v}{\alpha} \text{ [m], where:}$$

R_{min} – minimum radius of the vertical curve [m]
v – speed [km/h]

$$v = \begin{cases} v_p - \text{for road class Z and roads of lower classes} \\ v_m - \text{for road class G and roads of higher classes} \end{cases} \Rightarrow v = v_p = 60\frac{km}{h}$$

α – the angle of bend of the grade line [%]

$$R_{min} = \frac{100 \cdot 60}{4{,}306} = 1393{,}54m$$

Bend number	Mileage	Scheme	α [%]	R_{min} [m]
Z1 sag	0+230,00		4,306	1393,54
Z2 crest	0+639,88		2,914	2059,11
Z3 sag	1+330,00		1,147	5231,93
Z4 sag	2+380,00		0,729	8234,91
Z5 sag	2+718,36		2,386	2514,70

e) minimum radius due to the JoL16

For design speed $v_p = 60$ km/h:

- the smallest radius of the crest curve on a road with one carriageway $\mathbf{R_{min} = 2000\ m}$
- the smallest radius of the sag curve $\mathbf{R_{min} = 1500\ m}$

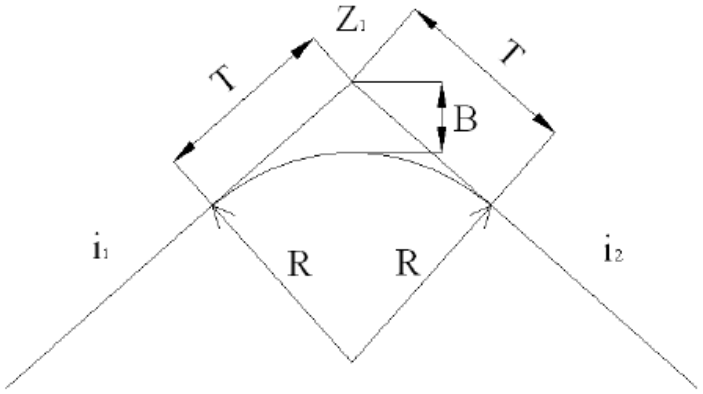
f) the adoption of values of the vertical curves

The list of calculations of the radiuses of the vertical curves

Bend number	Mileage	R_{min} [m] visibility	R_{min} [m] dynamics	R_{min} [m] aesthetics	R_{min} [m] JoL16	Adopted R [m]
Z1 sag	0+230,00	1681,97	554,40	1393,54	1500	**3500**
Z2 crest	0+639,88	4050,00	554,40	2059,11	2000	**4000***
Z3 sag	1+330,00	–	554,40	5231,93	1500	**7000**
Z4 sag	2+380,00	–	554,40	8234,91	1500	**15000**
Z5 sag	2+718,36	–	554,40	2514,70	1500	**3500**

* – due to the lack of ensuring the required visibility, it is necessary to apply the appropriate authority for a derogation from the Building Law

Values of the geometric parameters of the grade line:



a) tangent of the vertical curve

$$T = R \cdot tg\frac{\alpha}{2}\ [\mathrm{m}],$$ where:

T – tangent of the vertical curve [m]
R – radius of the vertical curve [m]
α – angle of bend of the grade line [%]

$$T = 3500 \cdot tg\frac{0{,}04306}{2} = 75{,}36m$$

b) external of the vertical curve

$$B = \frac{T^2}{2R}\ [\mathrm{m}],$$ where:

B – external of the vertical curve [m]
T – tangent of the vertical curve [m]
R – radius of the vertical curve [m]

$$B = \frac{75{,}36^2}{2 \cdot 3500} = 0{,}81m$$

External must be greater than 0.05 m (technological condition)

c) length of the vertical curve

$$Ł = 2 \cdot T\ [\mathrm{m}],$$ where

Ł – length of the vertical curve [m]
T – tangent of the vertical curve [m]

$$Ł = 2 \cdot 75{,}36 = 150{,}72m$$

List of characteristic values of the vertical curves

Curve no.	Mileage	R [m]	α [rad]	T [m]	B [m]	Ł [m]
Curve 1 sag	0+230,00	3500	0,04306	**75,36**	**0,81**	**150,72**
Curve 2 crest	0+639,88	4000	0,02914	**58,28**	**0,42**	**116,56**
Curve 3 sag	1+330,00	7000	0,01147	**40,14**	**0,12**	**80,28**
Curve 4 sag	2+380,00	15000	0,00729	**54,65**	**0,10**	**109,29**
Curve 5 sag	2+718,36	3500	0,02386	**41,76**	**0,25**	**83,51**

Curve no.	Mileage	T [m]	Mileage beginning of curve	Mileage end of curve
Curve 1 sag	0+230,00	75,36	0+154,64	0+305,36
Curve 2 crest	0+639,88	58,28	0+581,60	0+698,16
Curve 3 sag	1+330,00	40,14	1+289,86	1+370,14
Curve 4 sag	2+380,00	54,65	2+325,35	2+434,65
Curve 5 sag	2+718,36	41,76	2+676,60	2+760,12

Intermediate coordinates of the vertical curves:

a) coordinates of the grade line on the beginning and the end of vertical curve

$$H_{PŁ} = H_Z - i_n \cdot T \text{ [m]} \qquad H_{KŁ} = H_Z + i_{n+1} \cdot T \text{ [m]}, \text{ where:}$$

$H_{PŁ}$ – coordinates of the grade line on the beginning of vertical curve [m]
$H_{KŁ}$ – coordinates of the grade line on the end of vertical curve [m]
i_n – slope of the grade line before the bend [%]
i_{n+1} – slope of the grade line after the bend [%]
T – tangent of the vertical curve [m]

$H_{PŁ} = H_{BC}$; $H_{KŁ} = H_{EC}$

$$H_{PŁ} = 638{,}00 - (-0{,}03696) \cdot 75{,}36 = 640{,}79m$$
$$H_{KŁ} = 638{,}00 + 0{,}00610 \cdot 75{,}36 = 638{,}46m$$

Curve no.	H_Z [m]	i before bend [-]	i after bend [-]	T [m]	**$H_{PŁ}$ [m]**	**$H_{KŁ}$ [m]**
Curve 1 sag	638,00	-0,03696	0,00610	75,36	**640,79**	**638,46**
Curve 2 crest	640,50	0,00610	-0,02304	58,28	**640,14**	**639,16**
Curve 3 sag	624,60	-0,02304	-0,01157	40,14	**625,52**	**624,14**
Curve 4 sag	612,45	-0,01157	-0,00429	54,65	**613,08**	**612,22**
Curve 5 sag	611,00	-0,00429	0,01957	41,76	**611,18**	**611,82**

b) intermediate coordinates of the vertical curves

$$y_i = \pm \frac{x_i^2}{2 \cdot R} [m]$$

$$H_i = H_{mi} + y_i [m]$$

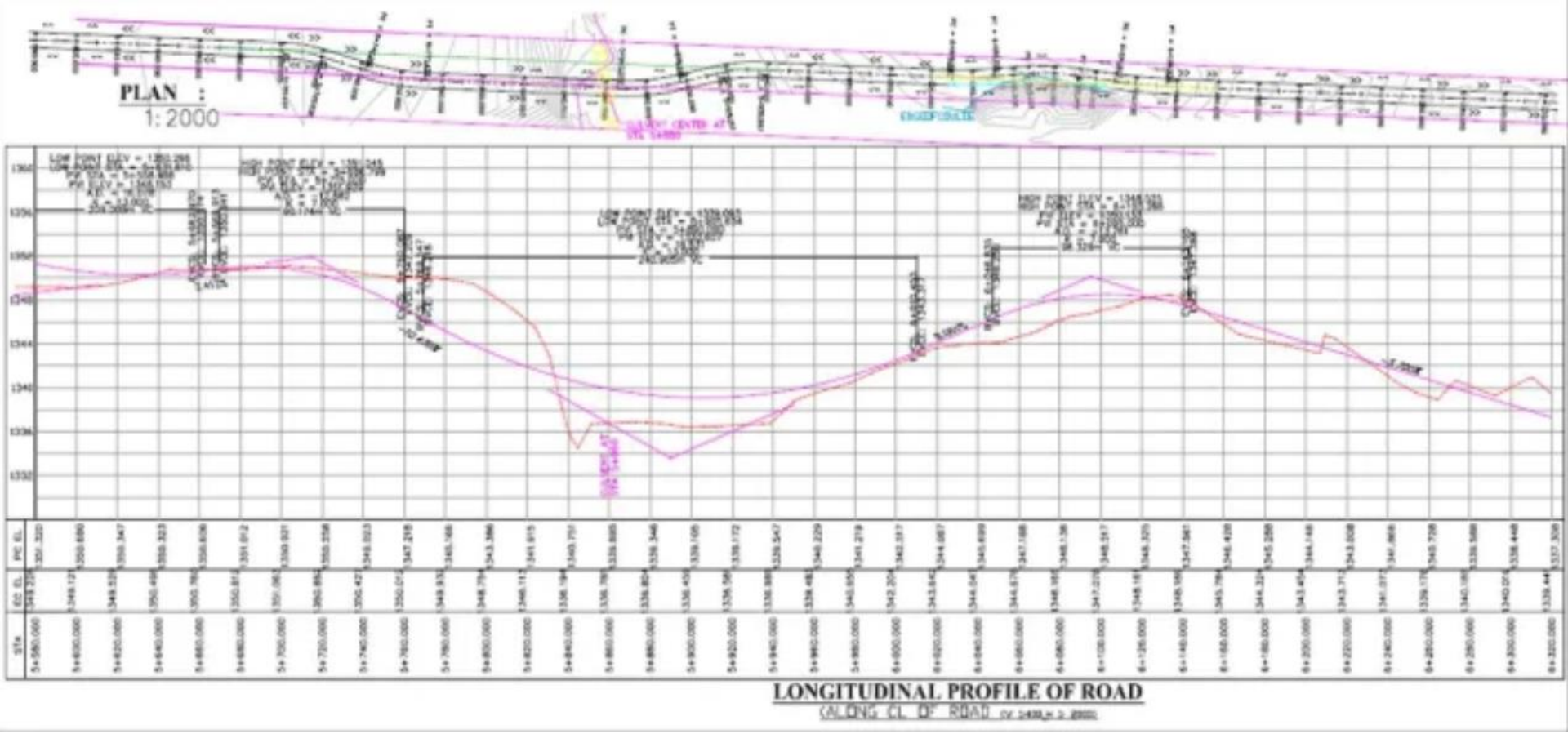
H_i – ordinate of the grade line after drawing the vertical curve [m]
H_m – ordinate of the grade line before drawing the vertical curve [m]
x_i – distance from the beginning of the vertical curve [m]
y_i – ordinate of the intermediate point on the vertical curve [m]
R – radius of the vertical curve [m]

$$y_i = -\frac{50^2}{2 \cdot 4000} = -0{,}31\,m$$

$$H_i = 640{,}45 + (-0{,}31) = 640{,}14\ m$$

For a crest curve we substitute ''+'' and for a sag curve ''-''.

Point	Mileage	x	y	H_m	H
BC	0+154,64	0	0,00	640,79	640,79
	0+164,64	10	0,01	640,42	640,43
	0+174,64	20	0,06	640,05	640,10
	0+184,64	30	0,13	639,68	639,80
	0+194,64	40	0,23	639,31	639,54
	0+204,64	50	0,36	638,94	639,29
	0+214,64	60	0,51	638,57	639,08
	0+224,64	70	0,70	638,20	638,90
Z1	0+230,00	75,36	0,81	638,00	638,81
	0+235,36	70	0,70	638,03	638,73
	0+245,36	60	0,51	638,09	638,61
	0+255,36	50	0,36	638,15	638,51
	0+265,36	40	0,23	638,22	638,44
	0+275,36	30	0,13	638,28	638,41
	0+285,36	20	0,06	638,34	638,39
	0+295,36	10	0,01	638,40	638,41
EC	0+305,36	0	0,00	638,46	638,46



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