Fundamentals of Road Construction

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Lecture 3

The subject of the lecture:

Horizontal alignment

Based on Polish legal acts and literature:

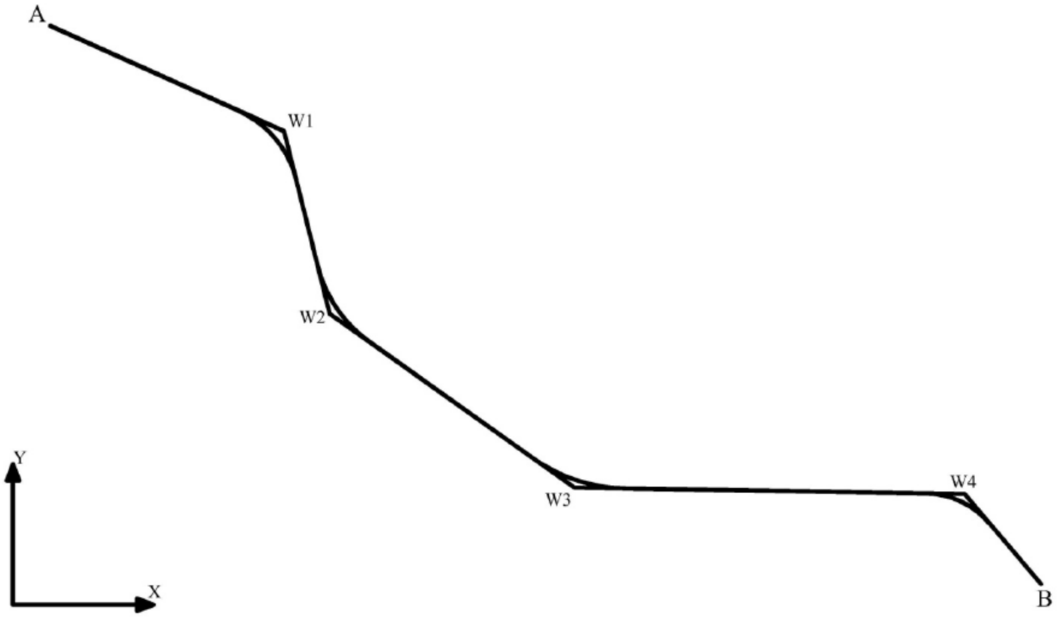
Regulation of the Minister of Infrastructure of June 24, 2022, on technical and construction regulations for public roads (Journal of Laws 2022, item 1518).

Rozporządzenie Ministra Infrastruktury z dnia 24 czerwca 2022 r. w sprawie przepisów techniczno-budowlanych dotyczących dróg publicznych (Dz.U. 2022 poz. 1518)

WR-D-22-2 Guidelines for the Design of Rural Road Sections, Part 2: Geometric Design

WR-D-22-2 Wytyczne projektowania odcinków dróg zamiejskich Część 2: Kształtowanie geometryczne

Coordinates of vertex points of horizontal alignment of the design road:



Point	Coordinates [m]	
	X	Y
A	90,00	1400,00
W_1	655,05	1145,05
W_2	765,65	704,15
W_3	1356,55	283,45
W_4	2300,07	268,08
B	2485,00	50,00

Distance (length) between vertex points:

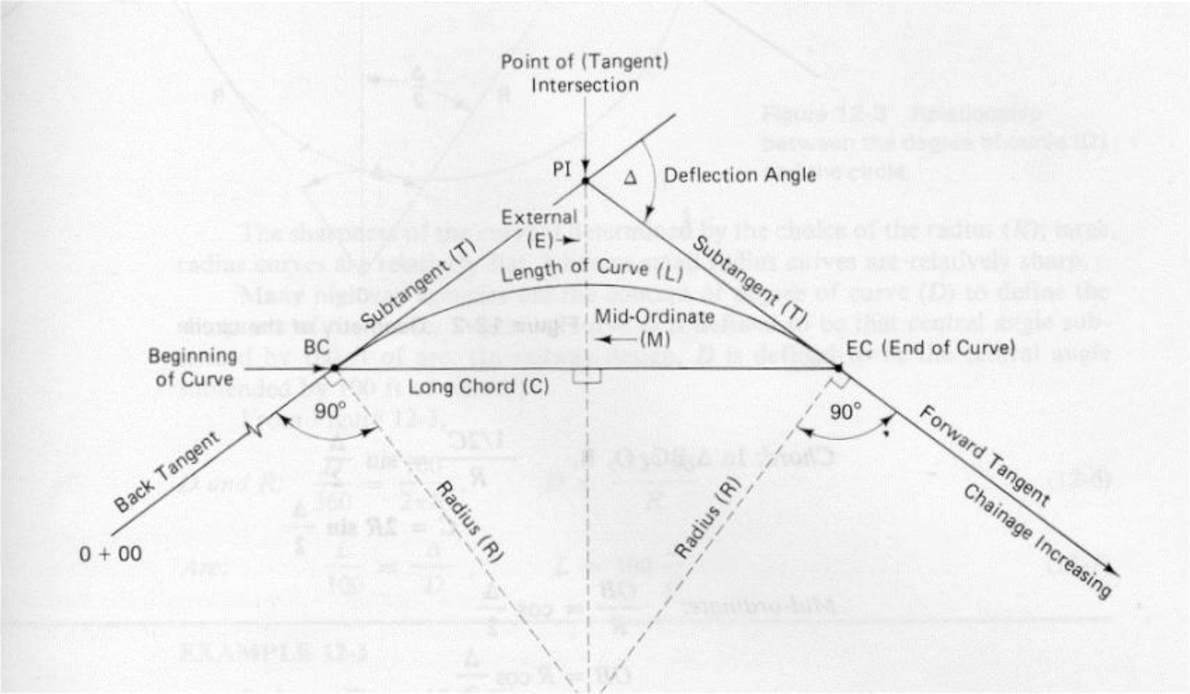
$$\boxed{\overline{AW_1} = \sqrt{(X_{W1} - X_A)^2 + (Y_{W1} - Y_A)^2}}$$

$$\overline{AW_1} = \sqrt{(655{,}05m - 90{,}00m)^2 + (1145{,}05m - 1400{,}00m)^2} = 619{,}90m$$

Section	Distance [m]
$\overline{AW_1}$	619,90
$\overline{W_1W_2}$	454,56
$\overline{W_2W_3}$	725,36
$\overline{W_3W_4}$	943,65
$\overline{W_4B}$	285,94
Σ	3029,41

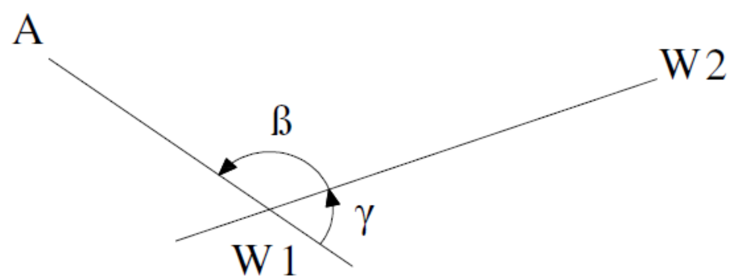
Deflection angle of horizontal alignment of the design road:

CIRCULAR HORIZONTAL CURVES



BC =Beginning of Curve	EC = End of Curve
PC = Point of Curve	PT = Point of Tangent
TC = Tangent to Curve	CT = Curve to Tangent

Source: https://www.cpp.edu/~hturner/ce220/circular_curves.pdf



$$\cos\beta_1 = \frac{\overline{AW_1}^2 + \overline{W_1W_2}^2 - \overline{AW_2}^2}{2 \cdot \overline{AW_1} \cdot \overline{W_1W_2}}$$

$$\gamma_1 = 180^\circ - \beta_1$$

$$\cos\beta_1 = \frac{(619{,}90)^2 + (454{,}56)^2 - (969{,}90)^2}{2 \cdot 619{,}90 \cdot 454{,}56} = -0{,}620696$$

$$\beta_1 = 128{,}3669^\circ$$

$$\gamma_1 = 180^\circ - 128{,}3669^\circ = 51{,}6331^\circ$$

	Deflection angle γ		
	[º]		[rad]
γ_1	51º37'59"	51,6331	0,901167
γ_2	40º28'06"	40,4684	0,706306
γ_3	34º30'59"	34,5163	0,602422
γ_4	48º46'09"	48,7691	0,851181

Tortuosity of section of the design road:

$$K = \frac{\sum_{i=1}^{n} |\gamma_n|}{L} \; [^\circ/\text{km}], \text{ where:}$$

K - tortuosity of the section of the design road [°/km]
$\Sigma\gamma_n$ - sum of the absolute deflection angles of horizontal alignment [°]
L - distance between vertices [km]
n - number of vertices [-]

$$K = \frac{51{,}6331^\circ + 40{,}4684^\circ + 34{,}5163^\circ + 48{,}7691^\circ}{3{,}03} = \frac{175{,}3867^\circ}{3{,}03}$$

$$K = 58\ ^\circ/\text{km}$$

The radius of a horizontal curve and the transverse slope can be:

1) calculated

$$R = \frac{V_{dp}^{\ 2}}{127(0{,}925 \cdot n \cdot f + 0{,}01 \cdot q)}$$

where:

R - radius of horizontal curve[m], the calculation result can be rounded, but not more than to ±5% of the value calculated from the formula,

V_{dp} - design speed [km/h],
127 - coefficient resulting from the conversion of units,
0,925 - coefficient taking into account the reduction of the coefficient of friction in the transverse direction in relation to the coefficient of friction in the longitudinal direction,

q - traversive slope [%]; generates values: as for example straight (with slope ≤ -2%) and from 2 to 7%,

n -unit coefficient of the friction coefficient f, permitted for use in the direction perpendicular to the drive [-]; the value n is assumed due to:

$$n = \begin{cases} 0{,}20 & \text{for } q \le -2\% \\ 0{,}06q - 0{,}02 & \text{for } q \ge 2\% \end{cases}$$

f - the nominal friction coefficient [-]; the value of f is taken from

$$f = -0{,}124 \ln(V_{dp}) + 0{,}8912$$

The radius of a horizontal curve and the transverse slope can be:

2) adopted according to the table

Relationship between the horizontal curve radius and the transverse slope

V_{dp} [km/h]	q [%]								
	-2,5	-2,0 [1]	2,0 [1]	2,5	3,0	4,0	5,0	6,0 [2]	7,0 [2]
	As in a straight section								
140	≥5 750	-	-	2 600	2 200	1 600	1 250	1 050	900
130	≥4 750	-	-	2 200	1 850	1 350	1 050	875	750
120	≥3 800	-	-	1 850	1 550	1 150	900	750	625
110	≥3 000	≥2 600	1 950	1 550	1 250	925	725	600	525
100	≥2 300	≥2 000	1 600	1 250	1 000	750	600	490	420
90	≥1 750	≥1 550	1 250	975	800	600	470	390	330
80	≥1 300	≥1 150	975	750	625	450	360	300	250
70	≥900	≥825	725	550	460	340	270	220	190
60	≥625	≥550	500	400	330	240	190	160	130
50	≥390	≥360	340	270	220	160	130	100	90
40	≥230	≥220	210	160	130	100	80	65	55
30	≥110	≥110	110	85	70	50	40	35	30

[1] A 2% transversive slope is not permitted on a class A, S or GP road or on a class G, Z, L or D road with a single-slope road section width exceeding 10.00 m (in accordance with subchapter 4.4.2)

[2] It is not recommended to use

Recommended minimum horizontal curve radius:

a) due to ride time

$$R_{min} = \frac{2 \cdot v_{dp}}{\gamma} \quad [m]$$

where:
V_{dp} - design speed [m/s]
γ - deflection angle of horizontal alignment [rad]

b) due to visibility at night

Minimum recommended radius of horizontal curve ensuring visibility of the path lane in the light of headlights

V_{dp} [km/h]	≥100	90	80	70	60	50	40	30
Minimum recommended radius of horizontal curve ensuring visibility of the path lane in the light of headlights [m]	1 600	1 400	1 100	650	400	230	130	60

Checking the requirements for the assumed radius of horizontal curves

a) roll-over stability condition

$$R_{\min} = \frac{v^2}{g \cdot \left(\frac{b}{2h} \pm i_0 \right)}, \ [\mathrm{m}] \ \text{where:}$$

v- speed $[\mathrm{m/s}]$

$$v = \begin{cases} v_p - \text{road of the class Z and lower (desgin speed)} \\ v_m - \text{road of the class G and upper (reliable speed)} \end{cases}$$

g- acceleration due to gravity $g \approx 9{,}81\ \mathrm{m/s^2}$

b- wheelbase vehicle (passenger car 1.5-1.8 m, lorry 2.0-2.4 m)

h- height of the center of gravity of the vehicle (passenger car 0.9-1.2 m, lorry 1.5-1.6 m)

i_0 - the transverse slope of the road on the curve [-]

$$R_{\min} = \begin{cases} \dfrac{v_p^{\ 2}}{g \cdot \left(\dfrac{b}{2h} - i_0\right)} [m] & \text{– two-side slop of the trafficway} \\ \dfrac{v_p^{\ 2}}{g \cdot \left(\dfrac{b}{2h} + i_0\right)} [m] & \text{– one-side slope of the trafficway} \end{cases}$$

$$R_{\min}{}^{(2)} = \frac{16{,}67^2}{9{,}81 \cdot (\frac{1{,}50}{2 \cdot 1{,}20} + 0{,}04)} = 42{,}60m$$

	i_o [%]	$R_{\min}{}^{(2)}$	
		+ i_o	**– i_o**
W1	4,0	**42,60**	—
W2	3,5	**42,92**	—
W3	3,0	**43,25**	—
W4	5,0	**41,97**	—

b) slip stability condition

$$\boxed{R_{\min} = \frac{v^2}{g \cdot (\varphi_R \pm i_0)}} \text{ [m], where:}$$

v - speed [m/s]

$$v = \begin{cases} v_p - \text{road of the class Z and lower (desgin speed)} \\ v_m - \text{road of the class G and upper (reliable speed)} \end{cases}$$

g - acceleration due to gravity $g \approx 9{,}81 \text{ m/s}^2$

φ_R - coefficient of transverse adhesion of the tire to the road

i_0 - the transverse slope of the road on the curve [-]

$$R_{\min} = \begin{cases} \dfrac{v_p^{\ 2}}{g \cdot (\varphi_R - i_0)} [m] & \text{– two-side slop of the trafficway} \\ \dfrac{v_p^{\ 2}}{g \cdot (\varphi_R + i_0)} [m] & \text{– one-side slope of the trafficway} \end{cases}$$

$\varphi_R = \mathbf{0{,}2}$ **[-]**

wet asphalt surface

$$R_{\min}^{(3)} = \frac{16{,}67^2}{9{,}81 \cdot (0{,}20 + 0{,}04)} = 118{,}03m$$

	i_o [%]	$\mathbf{R_{min}^{(3)}}$ **[m]**	
		+ i_o	**– i_o**
W1	4,0	**118,03**	—
W2	3,5	**120,54**	—
W3	3,0	**123,16**	—
W4	5,0	**113,31**	—

c) driving comfort condition

$$\boxed{R_{\min} = \frac{v^2}{g \cdot (\mu \pm i_0)}} \text{ [m], where:}$$

v - speed [m/s]

$$v = \begin{cases} v_p - \text{road of the class Z and lower (desgin speed)} \\ v_m - \text{road of the class G and upper (reliable speed)} \end{cases}$$

g- acceleration due to gravity $g \approx 9.81 \ \mathrm{m/s^2}$

μ- transverse acceleration factor [-]

i_0 - the transverse slope of the road on the curve [-]

$$R_{\min} = \begin{cases} \dfrac{{v_p}^2}{g \cdot (\mu - i_0)}[m] & - \text{ two-side slop of the trafficway} \\ \dfrac{{v_p}^2}{g \cdot (\mu + i_0)}[m] & - \text{ one-side slope of the trafficway} \end{cases}$$

It stands out due to the driving comfort:

- $\mu = 0{,}02$ – a passenger who does not observe the road, will not distinguish driving between section of straight or curved; the driver feels no tension
- $\mu = 0{,}06$ – a passenger has a poor feel of driving along the curvilinear section; the driver feels small tension
- $\mu = 0{,}10$ – a passenger feels the driving along the curvilinear section, but does not feel discomfort; the driver clearly feels the tension
- $\mu = 0{,}17$ – driving along a curvilinear section is inconvenient for everyone

adopted $\boldsymbol{\mu} = \mathbf{0{,}10}$ **[-]**, possibly $\mu = 0{,}12$ [-]

$$R_{\min}^{(4)} = \frac{16{,}67^2}{9{,}81 \cdot (0{,}10 + 0{,}04)} = 202{,}34m$$

	i_o [%]	$R_{\min}^{(4)}$	
		+ i_o	**– i_o**
W1	4,0	**202,34**	—
W2	3,5	**209,83**	—
W3	3,0	**217,90**	—
W4	5,0	**188,85**	—

JUXTAPOSITION R_{min}

	R (1)	**R_{min}** (2) roll-over	**R_{min}** (3) slip	**R_{min}** (4) comfort	i_o [%]	Adopted **R** [m]
W1	250	42,60	118,03	202,34	4,0	**250**
W2	320	42,92	120,54	209,83	3,5	**320**
W3	380	43,25	123,16	217,90	3,0	**380**
W4	200	41,97	113,31	188,85	5,0	**200**

Checking the ratio of horizontal curve radius to the length of the straight section

$L < 500$ m

where
L - length of the straight section

Checking the ratio of the radius of adjacent horizontal curves

Recommended maximum values of the ratio of radius adjacent horizontal curves

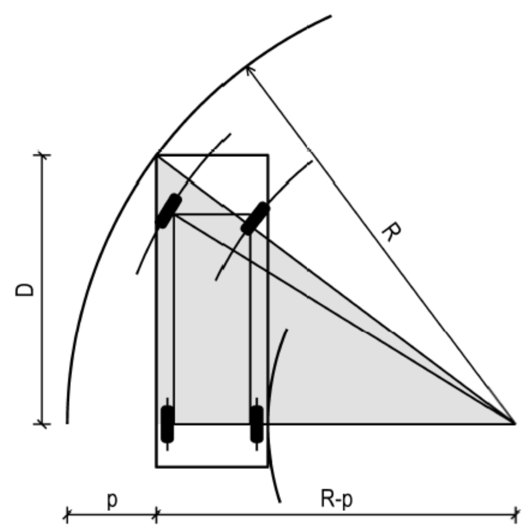
The value of the smaller radius (R_1) [m]	<300	300-799	800-1 500	>1 500
The largest ratio of radius adjacent horizontal curves ($R_2 : R_1$, $R_2 > R_1$)	1,5	2,0	2,5	facultative

Widening on the horizontal curve

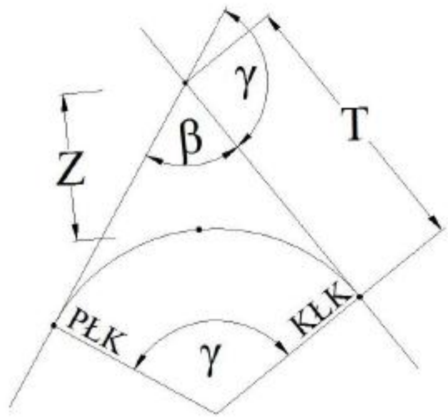
Formulas for calculating the amount of lane widening for typical size vehicles

typical size vehicle		D value [m]	Formula for calculating the amount of line widening p [m]
type	symbol		
passenger vehicle	PO	3,90	8/R
municipal vehicle	PK	6,50	20/R
two-axle bus	A2	9,70	50/R
three-axle bus	A3	10,60	60/R
truck with trailer	PN	-	

* in individual analyses for vehicles with an extended rear axle, the calculation assumes the position of the rear axle halfway between the extreme real rear axles



Calculation of basic elements of a horizontal curve



Signs:
PŁK – BC – begin of curve
KŁK – EC – end of curve
Z – E – external
T – tangent

Curve 1

$\gamma_1 = 51{,}6331^{\circ}$ $\qquad R_1 = 250$ m

– calculating the tangent of a curve

$$\boxed{T_1 = R_1 \cdot tg\,\frac{\gamma_1}{2} = 250 \cdot tg\,\frac{51{,}6331^{\circ}}{2} = 120{,}94\ m}$$

– calculating the external of a curve

$$Z_1 = \frac{R_1}{\cos\frac{\gamma_1}{2}} - R_1 = 250 \cdot \left(\frac{1}{\cos\frac{51{,}6331^{\circ}}{2}} - 1\right) = 27{,}72\ m$$

– curve length calculation

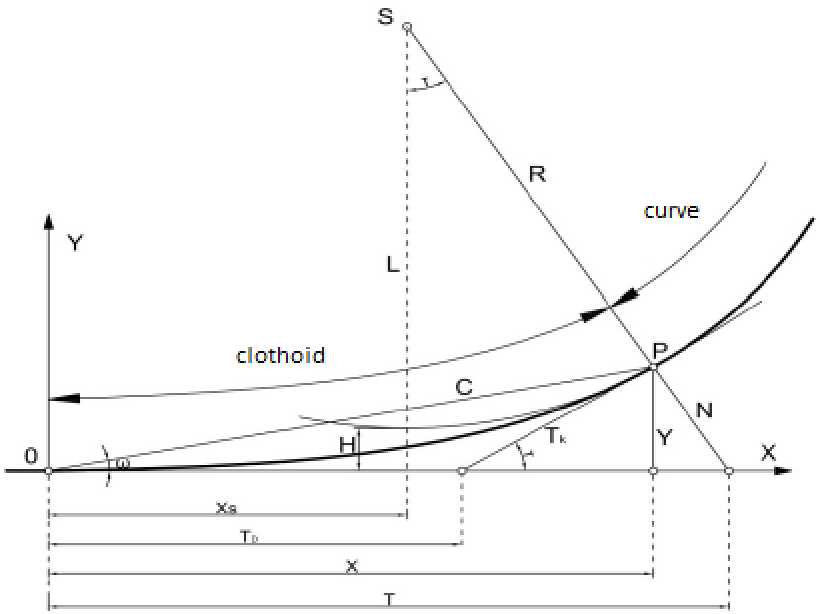
$$\boxed{D_1 = R_1 \cdot \frac{\pi}{180} \cdot \gamma_1 = 250 \cdot \frac{\pi}{180} \cdot 51{,}6331^{o} = 225{,}29\,m}$$

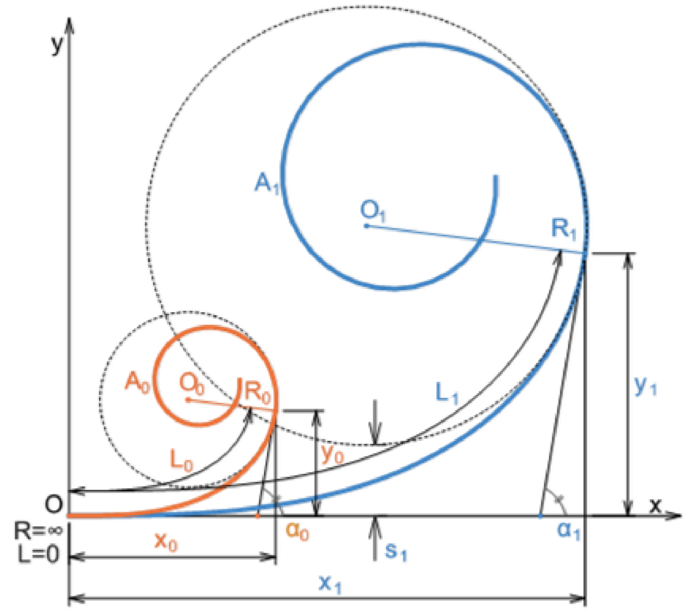
– widening on a curve

The widening is used when its value is greater than or equal to 0.2 m

	R [m]	γ [°]	T [m]	Z [m]	D [m]	$\frac{20}{R}$	p [m]
Curve 1	250	51,6331	120,94	27,72	225,29	0,08	–
Curve 2	320	40,4684	117,95	21,05	226,02	0,06	–
Curve 3	380	34,5163	118,05	17,91	228,92	0,05	–
Curve 4	200	48,7691	90,66	19,59	170,24	0,10	–

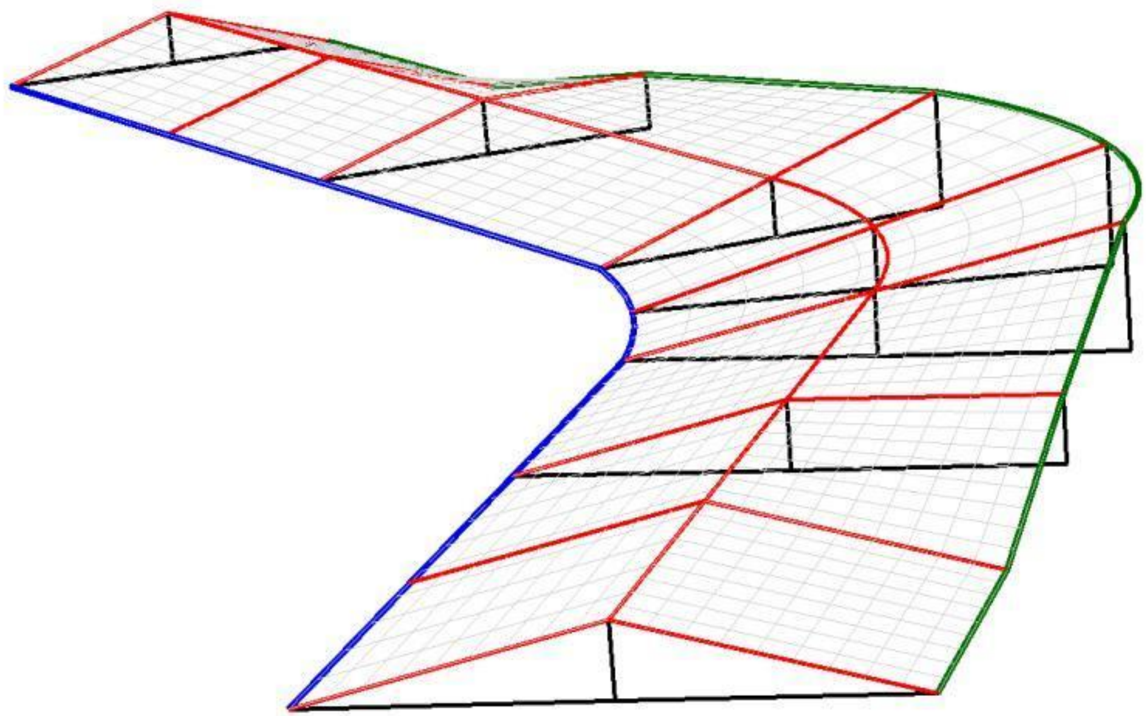
Determination of the clothoid a-parameter





Source: https://pwayblog.com/2016/07/03/the-clothoid/

The road ramp



Source: https://docplayer.pl/docs-images/64/51106521/images/11-1.jpg

Determination of the clothoid a-parameter

1) dynamics condition

$$a_{\min} = \sqrt{\frac{v^3}{k}}[m], \quad \text{where:}$$

a - clothoid a-parameter [m]

v - speed [m/s]

$v = v_p$ for all road classes $\Rightarrow v = v_p = 60\frac{km}{h} \Rightarrow v = 16{,}67\frac{m}{s}$

$k -$ permissible increase centripetal acceleration $\Rightarrow V_p = 60\frac{km}{h} \Rightarrow k = 0{,}7\frac{m}{s^3}$

$$a_{\min} = \sqrt{\frac{16{,}67^3}{0{,}7}} = 81{,}35m$$

Maximum increase in centripetal acceleration acting on the vehicle on the transition curve

V_{+} [km/h]	≥100	90	80	70	60	50	≤40
Maximum increase in centripetal acceleration acting on the vehicle on the transition curve [m/s2]	0,3	0,4	0,5	0,6	0,7	0,8	0,9

2) aesthetics condition

$$a_{min} = \frac{1}{3} R[m]$$

$a_{max} = R[m]$, where:

R- radius of the horizontal curve [m]

$$a_{1min} = \frac{1}{3} \cdot 200 = 66{,}67m$$

$$a_{1max} = 200m$$

	R [m]	a_{min} [m]	a_{max} [m]
curve 1	250	**83,33**	**250**
curve 2	320	**106,67**	**320**
curve 3	380	**126,67**	**380**
curve 4	200	**66,67**	**200**

3) construction of a road ramp condition

$$a_{\min} = \sqrt{\frac{R \cdot B}{2} \cdot \frac{i_n + i_o}{i_{d\max}}}\ \text{[m]},$$ where:

R – radius of the horizontal curve [m]

B – roadway width [m]; traffic line width outside the built-up area for a Z-class road is 2.75-3.00 m; adopted 6,00 m

i_o – the transverse slope of the roadway on a curvilinear section [-]

i_n – the transverse slope of the roadway on a straight section

i_d – additional slope of the roadway on the road ramp [-]

$$i_{d\min} \le i_d \le i_{d\max}$$

$$i_{d\min} = 0{,}1 \cdot \frac{B}{2} = 0{,}1 \cdot \frac{6{,}00}{2} = 0{,}3\%$$

$i_{d\max} = 1{,}6\%$ for design speed $v_P = 60 \frac{km}{h}$

$0{,}003 \le i_d \le 0{,}016$ adopted $i_d = 0{,}016$

Additional slope of the road edge

Design speed (km/h)	Additional slope of the road edge [%]	
	minimal on a section with a transverse slope	maximal
≥100	0,1a	0,9
80-90		1,0
60-70		1,6
≤50		2,0

a - distance of the edge from the axis of rotation

$$a_{1min} = \sqrt{\frac{250 \cdot 6{,}00}{2} \cdot \frac{0{,}02+0{,}04}{0{,}016}} = 53{,}03m$$

	R [m]	B [m]	i_n [%]	i_o [%]	i_d [%]	a_{min} [m]
curve 1	250	6,00	2,0	4,0	1,6	**53,03**
curve 2	320	6,00	2,0	3,5	1,6	**57,45**
curve 3	380	6,00	2,0	3,0	1,6	**59,69**
curve 4	200	6,40	2,0	5,0	1,6	**52,92**

4) widening of the roadway condition

calculated for the horizontal curves which have widening

$a_{min} = 1{,}86 \cdot \sqrt[4]{R^3 \cdot p_c}$ [m], where:

R – radius of the horizontal curve [m]
p_c – complete widening of the roadway on the curve [m]

$$a_{4min} = 1{,}86 \cdot \sqrt[4]{200^3 \cdot 0{,}4} = 78{,}67\ m$$

5) geometric condition

$a_{max} = R \cdot \sqrt{\gamma}$ [m], where:

R – radius of the horizontal curve [m]
γ – deflection angle of the horizontal curve [rad]

$$a_{1max} = 250 \cdot \sqrt{0{,}901167} = 237{,}32\ m$$

	R [m]	γ [rad]	a_{max} [m]
curve 1	250	0,901167	**237,32**
curve 2	320	0,706306	**268,93**
curve 3	380	0,602422	**294,94**
curve 4	200	0,851181	**184,52**

6) horizontal curve offset condition

recommended condition

for $H_{\min} = 0{,}50m \Rightarrow a_{\min} = 1{,}863 \cdot R^{\frac{3}{4}}$ [m],

for $H_{\max} = 2{,}50m \Rightarrow a_{\max} = 2{,}783 \cdot R^{\frac{3}{4}}$ [m], where:

H – horizntal curve offset [m]

$$a_{1\min} = 1{,}863 \cdot 250^{\frac{3}{4}} = 117{,}00m$$

$$a_{1\max} = 2{,}783 \cdot 250^{\frac{3}{4}} = 174{,}97m$$

	R [m]	a_{min} [m]	a_{max} [m]
curve 1	250	**117,00**	**174,97**
curve 2	320	**140,80**	**210,56**
curve 3	380	**160,17**	**239,53**
curve 4	200	**98,97**	**148,01**

7) proportionality condition

recommended condition

Signs:

L – Cl – the length of the clothoid

Ł – Cu – the length of the curve

$$\text{for } L{:}Ł{:}L = 1{:}4{:}1 \Rightarrow a_{min} = R \cdot \sqrt{\frac{\gamma}{5}} \ [m],$$

$$\text{for } L{:}Ł{:}L = 1{:}1{:}1 \Rightarrow a_{max} = R \cdot \sqrt{\frac{\gamma}{2}} \ [m], \ \text{gdzie:}$$

R- radius of the horizontal curve [m]

γ- deflection angle [rad]

$$a_{1min} = 250 \cdot \sqrt{\frac{0{,}901167}{5}} = 106{,}13m$$

$$a_{1max} = 250 \cdot \sqrt{\frac{0{,}901167}{2}} = 167{,}81m$$

	R [m]	γ [rad]	a_{min} [m]	a_{max} [m]
curve 1	250	0,901167	**106,13**	**167,81**
curve 2	320	0,706306	**120,27**	**190,17**
curve 3	380	0,602422	**131,90**	**208,55**
curve 4	200	0,851181	**82,52**	**130,47**

JUXTAPOSITION OF A-PARAMETER VALUES

	R [m]	a_{min} [m] dyn	a_{min} [m] asth	a_{min} [m] const	a_{min} [m] widening	a_{min} [m] offset	a_{min} [m] prop
curve 1	250	81,33	83,33	53,03	–	117,00	106,13
curve 2	320	81,33	106,67	57,45	–	140,80	120,27
curve 3	380	81,33	126,67	59,69	–	160,17	131,90
curve 4	200	81,33	66,67	52,92	78,67	98,97	82,52

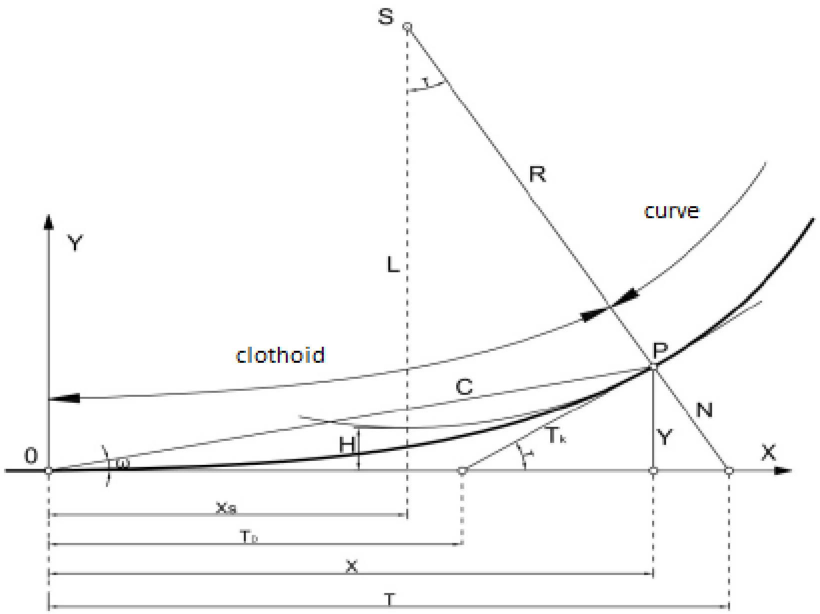
	R [m]	a_{max} [m] asth	a_{max} [m] geom	a_{max} [m] offset	a_{max} [m] prop
curve 1	250	250	237,32	174,97	167,81
curve 2	320	320	268,93	210,56	190,17
curve 3	380	380	294,94	239,53	208,55
curve 4	200	200	184,52	148,01	130,47

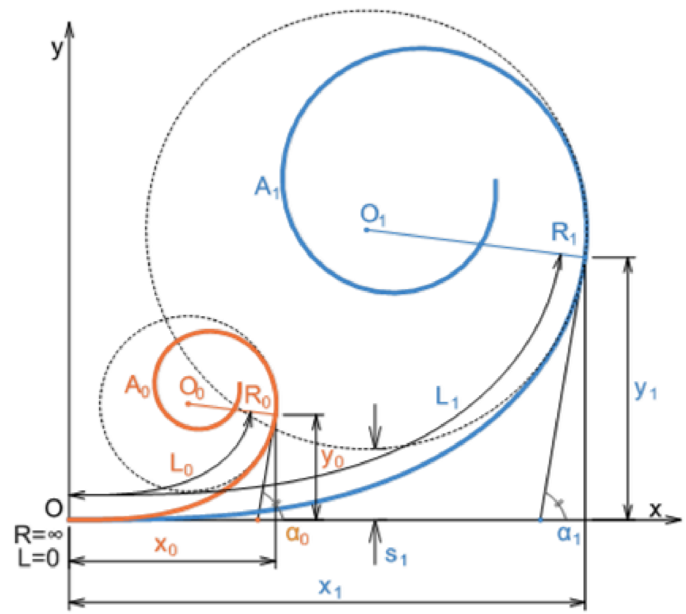
ADOPTION OF A-PARAMETER VALUE

$$a_{\min}^{(\max)} \leq a \leq a_{\max}^{(\min)}$$

	R [m]	$a_{min}^{(max)}$ [m]	$a_{max}^{(min)}$ [m]	**a [m]**
curve 1	250	117,00	167,81	**136,93**
curve 2	320	140,80	190,17	**154,92**
curve 3	380	160,17	208,55	**169,94**
curve 4	200	98,97	130,47	**109,55**

Determination of characteristic values of a colloidal transition curve





Source: https://pwayblog.com/2016/07/03/the-clothoid/

a) clothoid length

$$\boxed{L = \frac{a^2}{R}}\ [m],\ \text{where:}$$

L- length of the clothoid [m]
a- clothoid a-parameter [m]
R- radius of horizntal curve [m]

$$L = \frac{136,93^2}{250} = 75,00\,m$$

	R [m]	a [m]	**L [m]**
curve 1	250	136,93	**75,00**
curve 2	320	154,92	**75,00**
curve 3	380	169,94	**76,00**
curve 4	200	109,55	**60,00**

b) tangent angle at the end of the transition curve

$$\tau = \frac{L}{2 \cdot R} \quad [m]$$

$\tau_{min} = 3° \le \tau \le \tau_{max} = 30°$, where:

τ- tangent angle at the end of the transition curve [rad]
L- length of the clothoid [m]
R- radius of the horizontal curve [m]

$$\tau = \frac{75,00}{2 \cdot 250} = 0,150000 \quad rad$$

	R [m]	**L** [m]	τ [rad]	τ [°]	
curve 1	250	75,00	0,150000	8°35'40"	**8,5944**
curve 2	320	75,00	0,117188	6°42'52"	**6,7143**
curve 3	380	76,00	0,100000	5°43'46"	**5,7296**
curve 4	200	60,00	0,150000	8°35'40"	**8,5944**

For all transition curve the condition $\tau_{min} = 3° \le \tau \le \tau_{max} = 30°$ **is fulfilled**

c) coordinates of the end of the transition curve

$$X = L - \frac{L^5}{40a^4} + \frac{L^9}{3456a^8} - (...)[m] \qquad Y = \frac{L^3}{6a^2} - \frac{L^7}{336a^6} + \frac{L^{11}}{42240a^{10}} - (...)[m],$$ where:

X- coordinates of the end of the transition curve [m]

Y- coordinates of the end of the transition curve [m]

$$X = 75{,}00 - \frac{75{,}00^5}{40 \cdot 136{,}93^4} + (...) = 74{,}83m \qquad Y = \frac{75{,}00^3}{6 \cdot 136{,}93^2} - \frac{75{,}00^7}{336 \cdot 136{,}93^6} + (...) = 3{,}74m$$

	L [m]	**a** [m]	**X** [m]	**Y** [m]
curve 1	75,00	136,93	**74,83**	**3,74**
curve 2	75,00	154,92	**74,90**	**2,93**
curve 3	76,00	169,94	**75,92**	**2,53**
curve 4	60,00	109,55	**59,87**	**3,00**

d) coordinates of the center of horizontal curve

$X_s = X - (R \cdot \sin\tau)\ [m]$ $\qquad$ $Y_s = Y + (R \cdot \cos\tau)\ [m]$, where:

X_S- coordinates of the center of reduced horizontal curve [m]
Y_S- coordinates of the center of reduced horizontal curve [m]
τ- deflection angle of tangent on the end of the transition curve [rad]

$X_s = 74{,}83 - (250 \cdot \sin 0{,}150000) = 37{,}47m$ $\qquad$ $Y_s = 3{,}74 + (250 \cdot \cos 0{,}150000) = 250{,}94m$

	R [m]	**X** [m]	**Y** [m]	τ [rad]	$\mathbf{X_s}$ [m]	$\mathbf{Y_s}$ [m]
curve 1	250	74,83	3,74	0,150000	**37,47**	**250,94**
curve 2	320	74,90	2,93	0,117188	**37,48**	**320,73**
curve 3	380	75,92	2,53	0,100000	**37,99**	**380,63**
curve 4	200	59,87	3,00	0,150000	**29,98**	**200,75**

e) retraction of the horizontal curve

$$H = Y - R \cdot (1 - \cos\tau)\ [m]$$

$$H_{\min} = 0{,}5m \le H \le H_{\max} = 2{,}5m$$, where:

H- retraction of the horizontal curve [m]

τ- deflection angle of the tangent on the end of the horizontal curve [rad]

$$H = Y - R \cdot (1 - \cos\tau) = 3{,}74 - 250 \cdot (1 - \cos 0{,}150000) = 0{,}94m$$

	R [m]	**Y** [m]	**τ** [rad]	**H** [m]
curve 1	250	3,74	0,150000	**0,94**
curve 2	320	2,93	0,117188	**0,73**
curve 3	380	2,53	0,100000	**0,63**
curve 4	200	3,00	0,150000	**0,75**

For all transition curve the condition $H_{\min} = 0{,}5m \le H \le H_{\max} = 2{,}5m$ **is fulfilled**

f) external

$$N = \frac{Y}{\cos \tau} \ [\mathrm{m}], \text{ where:}$$

N- external of the transition curve [m]

τ- deflection angle of the tangent on the end of the horizontal curve [rad]

$$N = \frac{3,74}{\cos 0,150000} = 3,79 \ m$$

	Y [m]	τ [rad]	N [m]
curve 1	3,74	0,150000	**3,79**
curve 2	2,93	0,117188	**2,95**
curve 3	2,53	0,100000	**2,54**
curve 4	3,00	0,150000	**3,03**

g) short tangent

$$T_K = \frac{Y}{\sin \tau} \text{ [m], where:}$$

T_K- short tangent [m]

τ- deflection angle of the tangent on the end of the horizontal curve [rad]

$$T_K = \frac{3,74}{\sin 0,150000} = 25,05\ m$$

	Y [m]	τ [rad]	T_K [m]
curve 1	3,74	0,150000	**25,05**
curve 2	2,93	0,117188	**25,03**
curve 3	2,53	0,100000	**25,36**
curve 4	3,00	0,150000	**20,04**

h) long tangent

$$T_D = X - Y \cdot ctg\ \tau \quad [m],$$ where:

T_D- long tangent [m]

τ- deflection angle of the tangent on the end of the horizontal curve [rad]

$$T_D = 74,83 - 3,74 \cdot ctg\ 0,150000 = 50,06\ m$$

	X [m]	Y [m]	τ [rad]	T_D [m]
curve 1	74,83	3,74	0,150000	**50,06**
curve 2	74,90	2,93	0,117188	**50,04**
curve 3	75,92	2,53	0,100000	**50,69**
curve 4	59,87	3,00	0,150000	**40,05**

i) T_s section

$$T_S = (R + H) \cdot tg\frac{\gamma}{2} \quad [m], \text{ where:}$$

T_S- length of the section T_S [m]
γ- deflection angle of the horizontal curve [rad]

$$T_S = (250 + 0{,}94) \cdot tg\frac{0{,}901167}{2} = 121{,}40m$$

	R [m]	**H** [m]	γ [rad]	**T$_s$** [m]
curve 1	250	0,94	0,901167	**121,40**
curve 2	320	0,73	0,706306	**118,22**
curve 3	380	0,63	0,602422	**118,25**
curve 4	200	0,75	0,851181	**91,00**

j) integer tangent

$T_0 = X_s + T_S$ [m], where:

T_0- integer tangent [m]

$T_0 = 37{,}47 + 121{,}40 = 158{,}87$ m

	X_s [m]	T_s [m]	T_0 [m]
curve 1	37,47	121,40	**158,87**
curve 2	37,48	118,22	**155,71**
curve 3	37,99	118,25	**156,24**
curve 4	29,98	91,00	**120,98**

k) central angle of the reduced horizontal curve

$\boxed{\alpha = \gamma - 2 \cdot \tau}$ [m], where:

γ- deflection angle of the horizontal curve [rad]
τ- deflection angle of the tangent on the end of the horizontal curve [rad]
α- central angle of the reduced horizontal curve [rad]

$\alpha = 0{,}901167 - 2 \cdot 0{,}150000 = 0{,}601167$ rad

	γ [rad]	τ [rad]	α [rad]	α [^{o}]	
curve 1	0,901167	0,150000	0,601167	34°26'40"	**34,4443**
curve 2	0,706306	0,117188	0,471931	27°02'23"	**27,0397**
curve 3	0,602422	0,100000	0,402422	23°03'26"	**23,0571**
curve 4	0,851181	0,150000	0,551181	31°34'49"	**31,5803**

l) length of the reduced horizontal curve

$\boxed{Ł = R \cdot \alpha}$ [m], where:

α- central angle of the reduced horizontal curve [rad]

Ł- length of the reduced horizontal curve [m]

$Ł = 250 \cdot 0{,}601167 = 150{,}29$ m

	R [m]	α [rad]	**Ł** [m]
curve 1	250	0,601167	**150,29**
curve 2	320	0,471931	**151,02**
curve 3	380	0,402422	**152,92**
curve 4	200	0,551181	**110,24**

Coordinates for staking out the transition curve

Intermediate points for the transition curves in local coordinate systems

$$x(l) = l - \frac{l^5}{40a^4} + \frac{l^9}{3456a^8} - (\ldots)[m] \qquad y(l) = \frac{l^3}{6a^2} - \frac{l^7}{336a^6} + \frac{l^{11}}{42240a^{10}} - (\ldots)[m],$$ where:

$x(l)$ - local coordinate of the intermediate point of the transition curve [m]
$y(l)$ - local coordinate of the intermediate point of the transition curve [m]

where: $0 \leq l \leq L$

$$x(l) = 50{,}00 - \frac{50{,}00^5}{40 \cdot 136{,}93^4} + (\ldots) = 49{,}98m \qquad y(l) = \frac{50{,}00^3}{6 \cdot 136{,}93^2} - \frac{50{,}00^7}{336 \cdot 136{,}93^6} + (\ldots) = 1{,}11m$$

Transformation of the coordinate from the local system to the global system

$$\begin{cases} X = x \cdot \cos\alpha - y \cdot \sin\alpha + a \\ Y = x \cdot \sin\alpha + y \cdot \cos\alpha + b \end{cases}$$

Mileage of the horizontal alignment

		0+000,00	PA=PPT
$+AW_1$	+	619,90	
		0+619,90	W_1
$-T_{01}$	−	158,87	
		0+461,04	PKP_1
$+L_1$	+	75,00	
		0+536,04	KKP_1=$P\text{Ł}K_1$
$+\text{Ł}_1/2$	+	75,15	
		0+611,18	$\text{ŚŁ}K_1$
$+\text{Ł}_1/2$	+	75,15	
		0+686,33	$K\text{Ł}K_1$=KKP_1
$+L_1$	+	75,00	
		0+761,33	PKP_1
$-T_{01}$	−	158,87	
		0+602,45	W_1*
$+W_1W_2$	+	454,56	
		1+057,01	W_2
$-T_{02}$	−	155,71	
		0+901,31	PKP_2
$+L_2$	+	75,00	
		0+976,31	KKP_2=$P\text{Ł}K_2$
$+\text{Ł}_2/2$	+	75,51	
		1+051,82	$\text{ŚŁ}K_2$
$+\text{Ł}_2/2$	+	75,51	
		1+127,33	$K\text{Ł}K_2$=KKP_2
$+L_2$	+	75,00	
		1+202,33	PKP_2
$-T_{02}$	−	155,71	
		1+046,62	W_2*
$+W_2W_3$	+	725,36	
		1+771,98	W_3
$-T_{03}$	−	156,24	
		1+615,75	PKP_3
$+L_3$	+	76,00	
		1+691,75	KKP_3=$P\text{Ł}K_3$
$+\text{Ł}_3/2$	+	76,46	
		1+768,21	$\text{ŚŁ}K_3$
$+\text{Ł}_3/2$	+	76,46	
		1+844,67	$K\text{Ł}K_3$=KKP_3

$$W_1-W_1^* = 2\cdot T_0-2\cdot L-\text{Ł}$$
$$619{,}90-602{,}45 = 2\cdot 158{,}87-2\cdot 75{,}00-150{,}29$$
$$17{,}45 = 17{,}45$$

$$W_2-W_2^* = 2\cdot T_0-2\cdot L-\text{Ł}$$
$$1057{,}01-1046{,}62 = 2\cdot 155{,}71-2\cdot 75{,}00-151{,}02$$
$$10{,}40 = 10{,}40$$

$$W_3-W_3^* = 2\cdot T_0-2\cdot L-\text{Ł}$$
$$1771{,}98-1764{,}43 = 2\cdot 156{,}24-2\cdot 76{,}000-152{,}92$$
$$7{,}55 = 7{,}55$$

Signs:

PA – PPT – BDR – begin of the design road

W – V – vertex point

PŁK – BC – begin of the curve

KŁK – EC – end of the curve

PKP – BTC – begin of the transition curve

KKP – ETC – end of the transition curve

ŚŁK – CC – center of the curve

PB – KPT – EDR – end of the design road

L – the length of the clothoid

Ł – L’ – the length of the reducted curve

T_o – the length of the integer tangent

$+L_3$	+	76,00				
		1+920,67	PKP_3			
$-T_{03}$	–	156,24				
		1+764,43	W_3*			
$+W_3W_4$	+	943,65				
		2+708,08	W_4			
$-T_{04}$	–	120,98				
		2+587,10	PKP_4			
$+L_4$	+	60,00				
		2+647,10	KKP_4=$PŁK_4$	$W_4–W_4*$	=	$2·T_0–2·L–Ł$
$+Ł_4/2$	+	55,12		2708,08-2696,36	=	2·120,98-2·60,000-110,24
		2+702,22	$ŚŁK_4$	11,72	=	11,72
$+Ł_4/2$	+	55,12				
		2+757,34	$KŁK_4$=KKP_4			
$+L_4$	+	60,00				
		2+817,34	PKP_4			
$-T_{04}$	–	120,98				
		2+696,36	W_4*			
$+W_4B$	+	285,94				
		2+982,30	B			

Verification:

$AW_1 – (W_1–W_1*) + W_1W_2 – (W_2–W_2*) + W_2W_3 – (W_3–W_3*) + W_3W_4 – (W_4–W_4*) + W_4B =$ KT

619,90 – (619,90–602,45) + 454,56 – (1057,01–1046,62) + 725,36

– (1771,98–1764,43) + 943,65 – (2708,08–2696,36) + 285,94 = 2982,30

2982,30 = 2982,30

THANK YOU FOR YOUR ATTENTION